

Homework One - Version 1

Math 2030 - Elementary Probability

Due 31 May 2021

You wake up from your bed and there's a strange creature standing in your room. Before you can open your mouth the creature responds.

STRANGE CREATURE: Good morning beautiful human! Sorry for coming into your house unexpectedly, but you've been summoned by the high court of Probaland for your first homework assignment. I'll let you get ready and then we can go to the court together.

The creature leaves and you start getting ready for work. You would be surprised about the creature, but this type of thing happens frequently in your line of business. After putting on your Thursday outfit you tell the creature you're ready and you head off to the high court. On your way you notice it's a beautiful day in Probaland.

When you arrive one of the judges runs up to you.

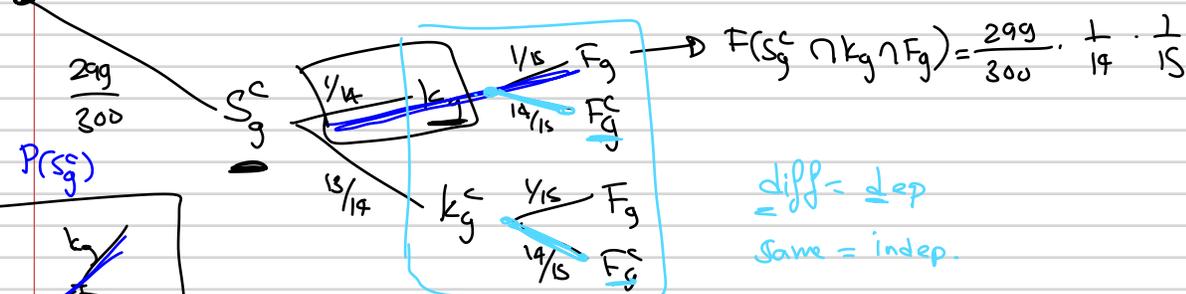
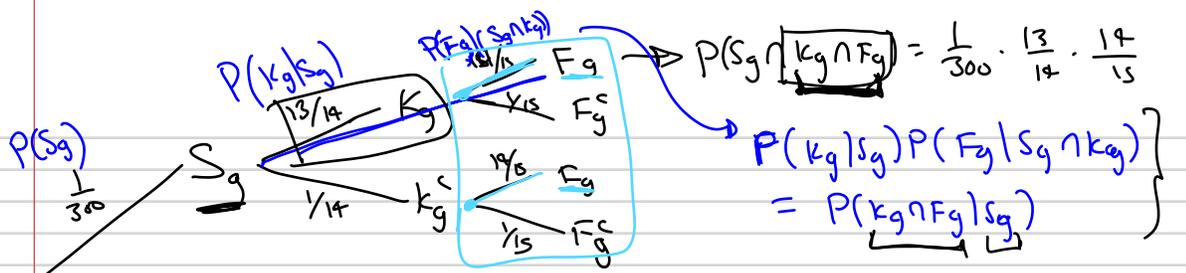
JUDGE: Oh my god. I'm so happy you arrived! We've been waiting a full 38 minutes and 22 seconds for your arrival. We need your help to solve a case. Come into my office and I'll go over the details.

After having walked into the judge's office, the judge explains the problem to you.

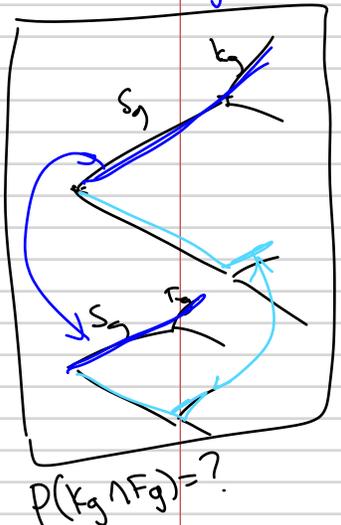
Question 1 We just had a massive trial of Sectaricus, I'm sure you saw the trial on the Probastream this morning, everyone was tuned in. As you know, there were exactly two witnesses: Kantonimus and Fumicious. They both have a long history of not being fully accurate. Kantonimus has a probability of $\frac{13}{14}$ of telling the truth and Fumicious has a probability of $\frac{14}{15}$ of telling the truth. Both Kantonimus and Fumicious stated that Sectaricus was guilty! We know that statistically Sectaricus is guilty with probability $\frac{1}{300}$. I'm not good at probability, but I heard you are! I was wondering if you could tell me what the probability that Sectaricus is actually guilty? (We can assume that Kantonimus and Fumicious telling the truth are independent events.)

~~Question 2 How long did it take you to complete this homework?~~

$$\begin{aligned}
 S_g &= \text{Sectaricus is guilty} \\
 P(S_g) &= \frac{1}{300} \\
 K_g &= \text{Kantonimus says Sect. is guilty} \\
 F_g &= \text{Fumicious says Sect. is guilty} \\
 P(S_g | K_g \cap F_g) &= \frac{P(S_g \cap (K_g \cap F_g))}{P(K_g \cap F_g)} \\
 &= \frac{P(S_g \cap K_g \cap F_g)}{P(\overline{S_g} \cap K_g \cap F_g) + P(S_g \cap K_g \cap F_g)}
 \end{aligned}$$



diff = dep
 same = indep.

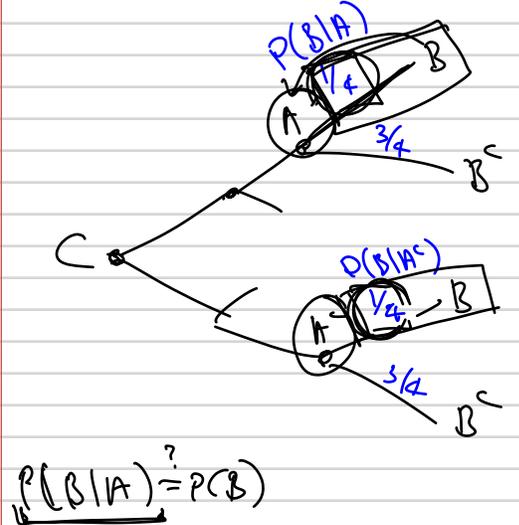


$$\begin{aligned}
 &= \frac{P(S_g \cap K_g \cap F_g)}{P(S_g^c \cap K_g \cap F_g) + P(S_g \cap K_g \cap F_g)} \\
 &= \frac{\frac{1}{300} \cdot \frac{13}{14} \cdot \frac{13}{15}}{\frac{299}{300} \cdot \frac{1}{14} \cdot \frac{1}{15} + \frac{1}{300} \cdot \frac{13}{14} \cdot \frac{13}{15}}
 \end{aligned}$$

$P(K_g) = ?$

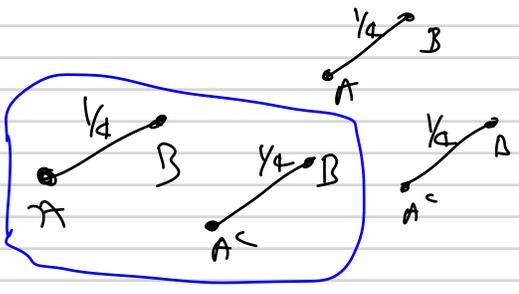
Law of total probability = 0.3784...

$$\begin{aligned}
 P(A) &= P(A \cap B) + P(A \cap B^c) & A &= K_g \cap F_g \\
 &= P(A|B)P(B) + P(A|B^c)P(B^c) & B &= S_g
 \end{aligned}$$



$$P(B|A) = P(B|A^c) = P(B) = \frac{1}{4}$$

A & B are ind.



$$P(B|A) = P(B)$$