$P(A)$ is the prior probability

* Event $\boxed{A}$ happens $\boxed{\text{\& then}}$ Event B happens

Probability of B

$$P(B|A) = \frac{P(A \cap B)}{\boxed{P(A)}}$$

$$\boxed{P(B|A)\,P(A) = \boxed{P(A \cap B)}} = P(B \cap A) = \boxed{P(A|B)\,P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \boxed{\frac{P(A|B)\,P(B)}{P(A)}}$$

$P(B|A)$ "likelihood of B given a fixed A"

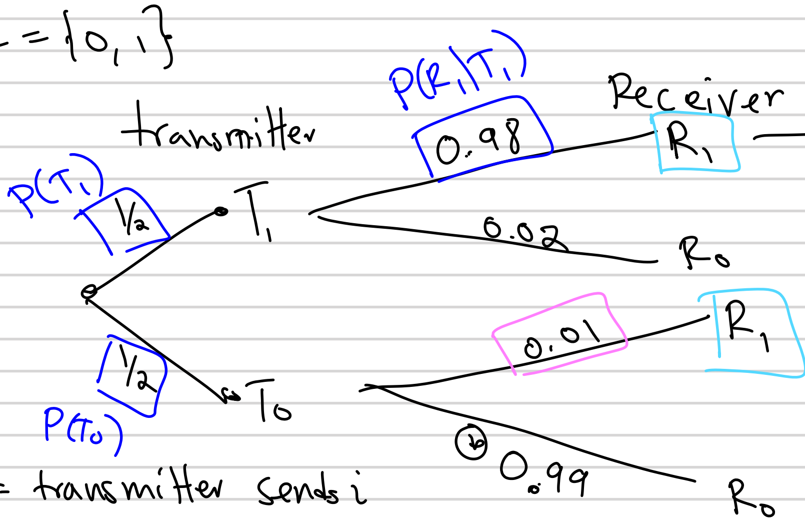
$P(A|B)$ "posterior probability of A given B"

$$P(A \cap B) = P(B|A)\,P(A)$$
$$= P(A|B)\,P(B)$$

Exercise 28 You're in charge of testing a new fibre optics cable that your company has manufactured. The cable transmits data from one end to the other. The transmitting side sends bursts of light which represent either a zero or a one to the receiving side. Since cables aren't perfect, there is some loss of data from one side to the other. You know that the probability that the receiving end receives a zero when the transmitter sends a zero is 99%. You know that the probability that the receiving end receives a one when the transmitter sends a one is 98%. You also know that the transmitting end sends a one exactly 50% of the time.

(1) What is the probability that the transmitter sent a 0 if the receiver received a 1?

$\Omega = \{0, 1\}$



$\longrightarrow p(R_1 \cap T_1) = P(R_1|T_1)P(T_1)$
$= 0.98 \cdot 0.5$

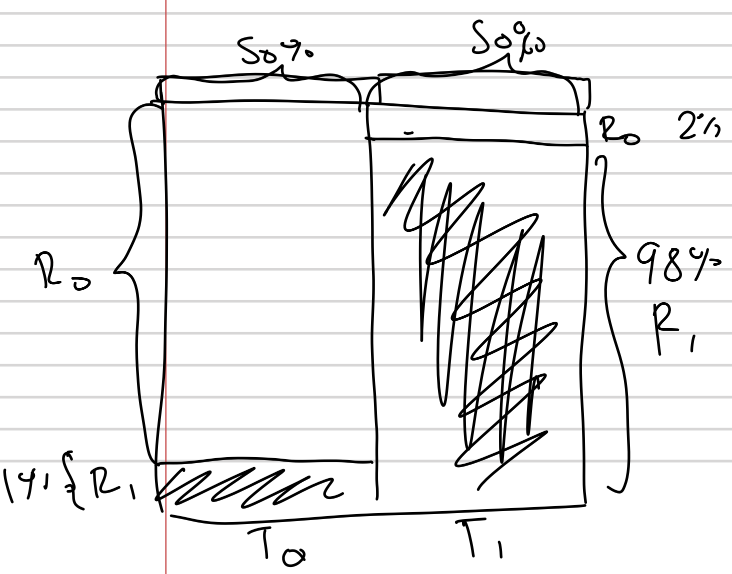
T_i = transmitter sends i
R_i = receiver receives i

$0.98 = P(R_1|T_1) = P(R_1)$
$0.01 = P(R_1|T_0) = P(R_1)$

(1) $P(T_0|R_1) = \dfrac{\boxed{P(T_0 \cap R_1)}}{P(R_1)}$

$= \dfrac{P(R_1|T_0)\,P(T_0)}{P(R_1|T_0)\,P(T_0) + P(R_1|T_1)P(T_1)}$

$= \dfrac{(0.01)(0.5)}{(0.01)(0.5) + (0.98)(0.5)} = \dfrac{1}{99} \approx 0.09259$



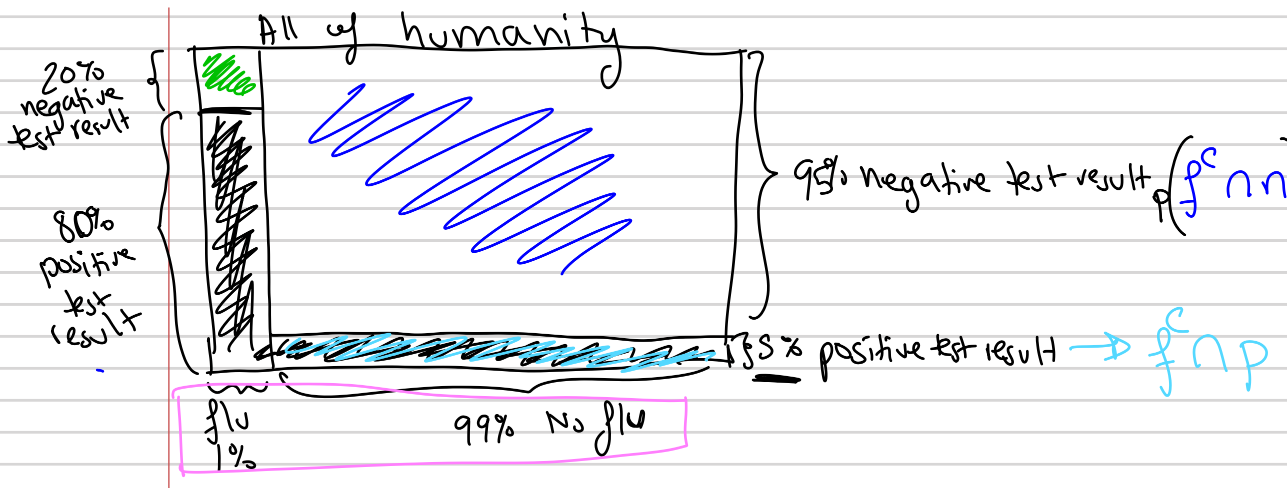
Exercise 29 You're not feeling to well one day and decide to go to the hospital to see if you have the flu. You know that, generally, $\boxed{1\%}$ of the population has the flu at any given time. You decide to take a flu test to see if you really have the flu or not. When you go into the hospital they say that a false positive (a person who isn't sick getting a positive result) happens about $\boxed{5\%}$ of the time and a false negative (a person who is sick gets a negative result) happens about 20% of the time. 0.1%

(1) What are the chances that the test will come back positive? (No matter if you have the flu or not)

(2) What are the chances that you have the flu and the test comes back negative?

(3) What are the chances that the test shows positive, but you don't have the disease?

(4) Is this a frequency or a subjective interpretation?



p = test is positive
n = test is negative $(= p^c)$

f = has the flu
f^c = does not have the flu

① $P(p) = p(f^c \cap p) + p(f \cap p)$

$= p(p|f^c)p(f^c) + p(p|f)p(f)$
$= (0.05)(0.99) + (0.8)(0.01) = 0.0575$

③ $P(f^c|p) = \frac{P(f^c \cap p)}{P(p)} = \frac{p(p|f^c)p(f^c)}{0.0575} = \frac{(0.05)(0.99)}{0.0575} = \boxed{0.8609}$

$\boxed{86.1\%}$ ✓

(1st test was positive)
⑤ What if we took a second test & it came back negative ~~positive~~.
What are the chances we don't have the flu

$p(f^c) = 0.8609$
$p(f) = 0.1391$

$p(f^c|n) = \frac{\boxed{p(n|f^c)p(f^c)}}{\cancel{P(n)}\ \boxed{p(n|f^c)p(f^c)} + \boxed{p(n|f)p(f)}} = \frac{(0.95)(0.8609)}{(0.95)(0.8609) + (0.2)(0.1391)}$

Dark blue greenbox

$= \frac{0.817855}{0.817855 + 0.02782}$

$= \frac{0.817855}{0.845675}$

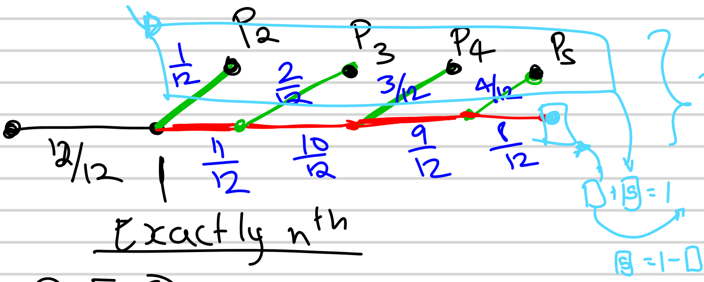
$= 0.9671$

$\boxed{96.7\%}$ ✓

diff

same

Exercise 25 How many people must be present in a room for there to be at least a 50% chance that two or more of them were born in the same month?



Exactly n^{th}

$P_1 = 0$

$P_2 = \boxed{\frac{12}{12}} \cdot \frac{1}{12} = \frac{1}{12}$

$P_3 = \frac{12}{12} \cdot \frac{11}{12} \cdot \frac{2}{12} = \frac{22}{144}$

$P_4 = \frac{12}{12} \cdot \frac{11}{12} \cdot \frac{10}{12} \cdot \frac{3}{12}$

$n = ?$

Less than or equal to n

$n=1$ $\quad 0$

$n=2$ $\quad \frac{1}{12} < \frac{1}{2}$

$n=3$ $\quad \frac{1}{12} + \frac{22}{144}$

$= 1 - \frac{12}{12} \cdot \frac{11}{12} \cdot \frac{10}{12}$

$= \frac{17}{72} < \frac{1}{2}$

$n=4$ $\quad P_1 + P_2 + P_3 + P_4$

$= 1 - \frac{12}{12} \cdot \frac{11}{12} \cdot \frac{10}{12} \cdot \frac{9}{12}$

$= \frac{123}{288} < \frac{1}{2}$

$\boxed{n=5}$ $P_1 + P_2 + P_3 + P_4 + P_5$

$= 1 - \frac{12}{12} \cdot \frac{11}{12} \cdot \frac{10}{12} \cdot \frac{9}{12} \cdot \frac{8}{12}$

$\boxed{= \frac{178}{288}} > \frac{1}{2}$

$P_1 + P_2 + P_3 + \cdots + P_{23}$

$= 1 - \_\_\_\_\_$