homework

① How to convert English to $\rightarrow$ Probability?

② How to attack a problem?

converts or to exclusive or

Exercise 18 Suppose we are looking at a group of humans that are either right or left handed and with either green or blue hair. If 88% of blue-haired individuals are right handed and 78% of green-haired people are right handed then which of the following are true? Which are false? Which can't be decided with the given information?

$P(\text{Right hand}) \neq 0.83$

Not

(1) The overall proportion of right handers in this group is exactly 83%. Can't be decided

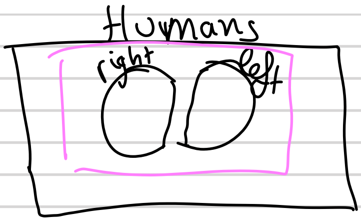
"given Everything"

(2) The overall proportion of right handers in this group is between 88 and 78 percent.

(3) If the ratio of blue-haired humans to green-haired humans is 1 to 1 then the overall proportion of right handers in this group is exactly 83%.

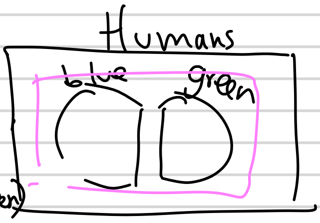
(1) What do I know?

What do I want?



$P(\text{right handed} \mid \text{Blue-haired}) = 0.88$

$P(\text{right-handed} \mid \text{green-haired}) = 0.78$



$P(\text{right-handed}) = P(\text{Right} \cap \text{Blue}) + P(\text{Right} \cap \text{Green})$
$= P(\text{Right} \mid \text{Blue})\boxed{P(\text{Blue})} + P(\text{Right} \mid \text{Green})\boxed{P(\text{Green})}$

$0 \leq P(\text{Blue}) \leq 1$
$0 \leq P(\text{Green}) \leq 1$

$P(\text{Blue}) = 1 \Rightarrow P(\text{Right-handed}) = P(R \mid B)P(B) + P(R \mid G)P(G)$

Either Green or Blue $\Rightarrow$ $P(G) = 1 - P(B) = P(B^c) = 0$

$= 0.88(1) + (0.78)(0) = 0.88$

$P(B) = 0$
$P(G) = 1$

$\Rightarrow P(R) = P(R \mid B)\,P(B) + P(R \mid G)\,P(G)$
$= 0.88 \cdot 0 + 0.78 \cdot 1$
$= 0.78$

$P(\text{Right} \cap \text{Blue}) =$ "If 88% of humans are right-handed & bluehaire

Exercise 23 Suppose I have a deck of cards where

- 15% are red on both sides $P(\text{red on both sides}) = 0.15$
- 65% are red on one side and black on the other $\boxed{P(\text{red on one side black on other})} = 0.65$
- 20% are black on both sides $P(\text{black on both sides}) = 0.2$

n

0.6 $\frac{1}{2}$ $P(\text{red on top, black on bottom})$

0.4 $\frac{1}{2}$ $P(\text{red on bot, black on top})$

I've randomly shuffled the deck and the top card is red on top. What is the probability that the other side of the top card is black?

choosing card all have same prob

Since I can think of cards with flipping, I have "2n" cards.

$$P(\text{bottom side is black} \mid \text{top side is red})$$

"And"

$$= \frac{P(\text{bottom side is black} \cap \text{top is red})}{P(\text{top side is red})}$$

card flipped either way

$$= \frac{0.65 \cdot \boxed{0.5}}{0.65 \cdot \boxed{0.5} + 0.15} = \frac{0.325}{0.475} = 0.6842$$

0.6 0.6

$$\cancel{\frac{0.65}{0.65 + 0.15 \times \frac{1}{2}}}$$

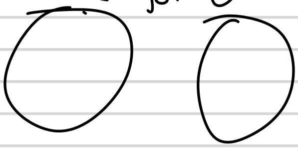
$$\frac{1/2}{1/2} \sim \frac{0.65 \cdot 0.5}{0.65 \cdot \frac{1}{2} + 0.15}$$

$$0.65 + 0.3 + 0.4 > 1$$
$$(0.15 \times 2) \quad (0.2 \times 2)$$

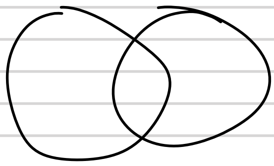
Exercise 22 Suppose that we are dealt two cards from a standard deck of 52 cards. What is the probability that the second card is a diamond knowing that the first one is red?

Exercise 23 Suppose I have a deck of cards where

disjoint



?



A & B are disjoint

$A \cap B = \emptyset$

$P(A \cap B) = 0$

A & B are independet

$A \cap B = ?$

$P(A \cap B) = P(A)P(B)$

Exercise 24 Suppose that (a) two independent internet companies supply internet to your neighbourhood. To save money they (b) only supply internet sometimes. The (c) first company supplies internet with probability 0.2 and the (d) second with probability 0.7. (e) If both companies supply internet then the whole neighbourhood has internet with probability 1. (f) If only one company supplies internet then the whole neighbourhood has internet with probability 0.4. (g) If none supply internet, then there is no internet.

(1) What is the probability of both companies supplying internet? $P(A\cap B)$

(2) What is the probability of exactly one company supplying internet? $P(A\cap B^c)+P(A^c\cap B)$

(3) What is the probability of no company supplying internet?

(4) What is the probability that your neighbourhood has internet? $\boxed{P(C)} = \boxed{P(C\cap(A\cap B))+P(C\cap(A^c\cap B)) + P(C\cap(A\cap B^c)) + P(C\cap(A^c\cap B^c))}$

$P(C\mid A\cap B)P(A\cap B)$

(a) A = 1st company supplies internet
B = 2nd " "
$P(A\cap B) = P(A)P(B)$

$A = A_1 \sqcup A_2$
$P(A) = P(A_1)+P(A_2)$

Non-ind:
$P(A\cap B) = P(A\mid B)\,P(B)$

(b) $P(A)<1$
$P(B)<1$

(c) $P(A)=0.2$

(d) $P(B)=0.7$

(e) C = whole neighborhood / has internet
$P(C\mid A\cap B)=1$

(f) $P(C\mid A\cap B^c)+P(C\mid A^c\cap B) = 0.4$

(g) $P(C\mid A^c\cap B^c) = 0$

(1) $P(A\cap B) \overset{(a)}{=} P(A)P(B) \overset{(c),(d)}{=} 0.2\cdot 0.7 = 0.14$

(2) $P(A\cap B^c)+P(A^c\cap B) \overset{(a)}{=} P(A)P(B^c)+P(A^c)P(B)$ $\quad P(A^c)=1-P(A)$

$\overset{(c),(d)}{=} (0.2)(0.3) + (0.8)(0.7)$

$= 0.62$

$P((A\cup B)\setminus(A\cap B)) = P(A\cup B) - P((A\cap B)\cap(A\cup B))$

$= \boxed{P(A\cup B)} - \boxed{P(A\cap B)}$

$= P(A)+P(B)-P(A\cap B) - P(A\cap B)$

$= 0.2 + 0.7 - 0.14 - 0.14$

$= 0.9 - 0.28 = 0.62$

incl-excl

$P(A\cup B) = P(A)+P(B)-P(A\cap B)$

(4) $(A\cap B)\sqcup(A^c\cap B)\sqcup(A\cap B^c)\sqcup(A^c\cap B^c)$

$P(A) = P(A\mid A_1)P(A_1) + P(A\mid A_2)P(A_2) + \cdots$

Exercise 26 Suppose you roll a fair six sided dice until you roll a number that's already been rolled.

(1) Let p_i to be the probability that you rolled *exactly* i times before succeeding. Calculate p_i for the numbers 1 until 10.

(2) Without any calculations find $p_1 + p_2 + \cdots + p_{10}$ and explain why you found that solution.

ⓐ $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$

ⓑ stopping condit^n

ⓒ p_i = P(rolled exactly i times before rolling a number already rolled)

= P(the i^{th} roll is equal to a previous roll & all other rolls are diff)

① ⓐ p_1 = P(the 1^{st} roll is equal to a previous roll)

$= 0$ (b/c there is no prev. roll).

ⓑ p_2 = P(the 2^{nd} roll is equal to a prev roll & all other rolls are diff)

$= \frac{6}{6}\frac{1}{6}$ = P(1^{st} roll #)

ⓒ p_3 = P(rolled exactly 3 times before succeeding)

$= \frac{6}{6}\frac{5}{6} \cdot \frac{2}{6} = \frac{10}{36}$ — 3^{rd} roll has to match 1^{st} or 2^{nd} roll

2^{nd} has to be diff

ⓓ $p_4 = \frac{6}{6}\frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} = \frac{60}{216}$

2^{nd} (diff) 3^{rd} roll (diff) 4^{th} roll (same)

ⓔ $p_5 = \frac{6}{6}\frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{4}{6} = \frac{2\cancel{1}0}{1296}$

2^{nd} roll (diff) 3^{rd} roll (diff) 4^{th} roll (diff) 5^{th} roll (same)

ⓕ $p_6 = \frac{6}{6}\frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{5}{6} = \frac{600}{7776}$

ⓖ $p_7 = \frac{6}{6}\frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{1}{6} \cdot \frac{6}{6} = \frac{120}{7776}$

2^{nd} 3^{rd} 4^{th} 5^{th} 6^{th}

ⓗ $p_8 = 0$ $\frac{6}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{1}{6} \cdot \frac{0}{6} \cdot \frac{6}{6} = 0$

$p_9 = 0$

$p_{10} = 0$

Ex

1^{st} roll

p_2 asks probability that 2^{nd} roll is equal to 5 — $\frac{1}{6}$

p_3 asks probability that 2^{nd} roll is not equal to 5 & 3^{rd} roll is equal to 5 or 2^{nd} roll

② $p_1 + p_2 + p_3 + \cdots + p_{10} = 1$ since by 7^{th} roll we must have 2 dice that are same.

Exercise 30 You and your nine friends are playing truth or dare! In order to decide who starts you decide to take 10 strips of paper of equal length and cut one of them shorter. The first person to pick the shorter one gets to start. Is this a fair way of choosing who begins? (In other words, does everyone have the same probability of choosing the short strip of paper?)