16, 17.3, 14(13), Dist.(15)

20 May 2021

16) Show that

$$\to P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+\boxed{P(A\cap B\cap C)}$$

Thm 6.1 (complement rule) – $P(A^c)=1-P(A)$

Thm 6.2 (Diff. rule) – $P(B\cap A^c)=P(B\setminus A)=P(B)-P(A\cap B)$

→ Thm 6.3 (Inc-exc. rule) – $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

Show that

inc-exc

$$P(A\cup(B\cup C)) = P(A)+P(B\cup C)-P(A\cap(B\cup C))$$

inc-exc

$$=P(A)+(P(B)+P(C)-P(B\cap C))-P(A\cap(B\cup C))$$

distribution

* dist

$$=P(A)+P(B)+P(C)-P(B\cap C)-P((A\cap B)\cup(A\cap C))$$

inc-exc

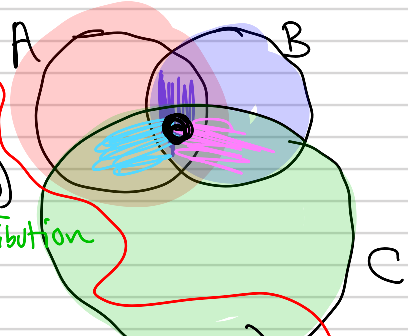
$$=P(A)+P(B)+P(C)-P(B\cap C)-(P(A\cap B)+P(A\cap C)-P((A\cap B)\cap(A\cap C)))$$

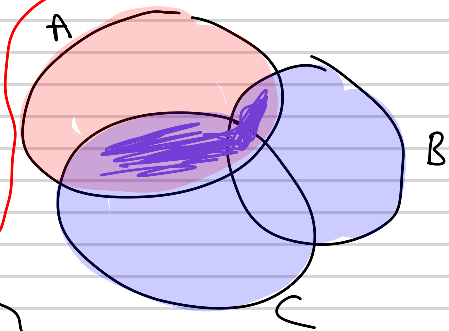
* dist (of -)

$$=P(A)+P(B)+P(C)-P(B\cap C)-P(A\cap B)-P(A\cap C)+\boxed{P((A\cap B)\cap(A\cap C))}$$

* equality of sets

$$= \qquad +P(A\cap B\cap C)$$





Too much

$$(A\cap B)\cap(A\cap C) \overset{1}{=} A\cap(B\cap A)\cap C$$
$$\overset{2}{=} A\cap(A\cap B)\cap C$$
$$\overset{3}{=} (A\cap A)\cap(B\cap C)$$
$$\overset{4}{=} A\cap(B\cap C)$$
$$\overset{5}{=} A\cap B\cap C$$

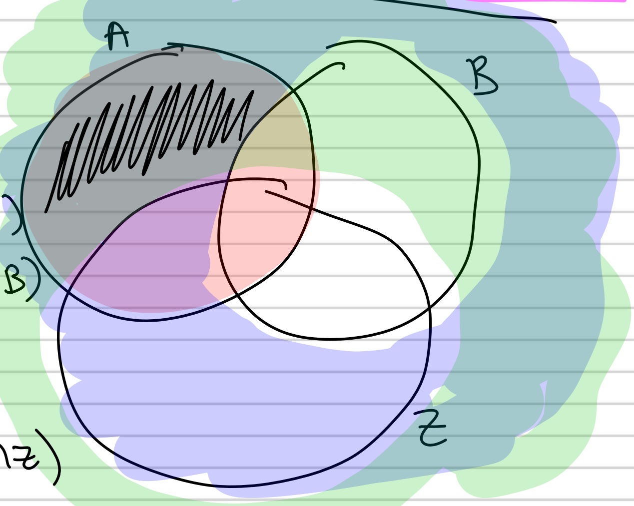
De Morgan's Laws

$$(A\cup B)^c = A^c\cap B^c$$
$$(A\cap B)^c = A^c\cup B^c$$

(17.3) $P(A)=0.8 \quad P(B)=0.4 \quad P(Z)=0.3 \quad P(A\cap B)=0.2 \quad P(A\cap Z)=0.2 \quad P(B\cap Z)=0.1$

$\boxed{P(A\cap B\cap Z)=0}$

$P(\underbrace{A\cap B^c\cap Z^c})$



complement rule) − $P(A^c)=1-P(A)$

(Diff. rule) − $P(B\cap A^c)=P(B\setminus A)=P(B)-P(B\cap A)$

(Inc-exc. rule) - $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$P((A\cap B^c)\cap Z^c) \overset{\text{Diff rule}}{=} P(A\cap B^c)-P((A\cap B^c)\cap Z)$$

$$\overset{\text{Diff rule}}{=} P(A)-P(A\cap B)-P(\underline{A\cap B^c\cap Z})$$

$$\overset{\text{comm.}}{=} P(A)-P(A\cap B)-P((A\cap Z)\cap B^c)$$

$$\overset{\text{diff rule}}{=} P(A)-P(A\cap B)-\left(P(A\cap Z)-P(A\cap Z\cap B)\right)$$

$$\overset{\text{dist.}}{=} P(A)-P(A\cap B)-P(A\cap Z)+P(A\cap B\cap Z)$$

$$= 0.8-0.2-0.2+0$$

$$= 0.8-0.4$$

$$\boxed{=0.4}$$

$A\cap B^c\cap Z$
$\parallel$
$A\cap Z\cap B^c$

14 $\boxed{\Omega = \{0, 1, 2\}}$ flip coin twice

0 heads, 1 head, 2 heads

(1) lands heads twice = 2 heads

yes, $\{2\}$ = A

(2) lands heads at least once = ≥ 1 head

yes, $\{1, 2\}$ = B

(3) lands on heads & then lands on tails.

No, b/c our sample doesn't order coins.

order

(4) lands on heads once & lands on tails once = 1 head

yes, $\{1\}$ = $A \cap B$

$\underbrace{(H,T) \text{ or } (T,H)}_{\text{represent 1 heads}}$

$(H,T) \neq \{$1 Head, 1 tails$\}$

= ?

Distribut^{n}s

~~Ω = sample space~~

Distribution over S (S = sample space) is a function $P: S \to \mathbb{R}$

For all $A \subseteq S$

- $P(A) \geq 0$ (non-negative)
- if $A = A_1 \sqcup A_2 \sqcup \cdots \sqcup A_n$

  $P(A) = P(A_1) + P(A_2) + \cdots + P(A_n)$ (Addit^n)
- $P(S) = 1$

$A \sqcup B$ is union where $A \cap B = \phi$

Well defined distribution $P(A) = \frac{|A|}{|S|}$

① Bernoulli Distribution

$S = \{x, y\}$ $P(x) = p$ $P(y) = 1 - p$ $P(S) = 1$ $P(\phi) = 0$

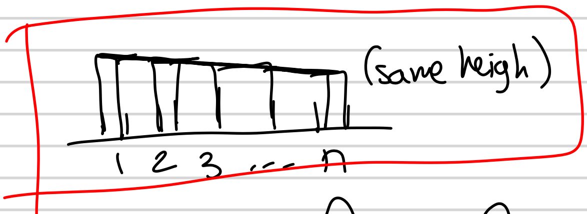
$p \in [0, 1]$

② Discrete Uniform distribution

uniform: probability of all thing is equal.

$S = \{1, 2, 3, \ldots, n\}$

$P(1) = P(2) = P(n) = \frac{1}{n}$

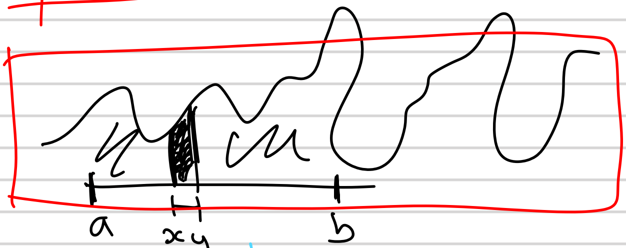


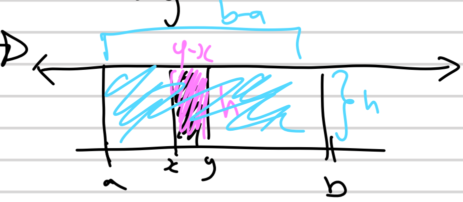
$A = \{1, 2\}$ $\frac{|A|}{n} = \frac{2}{n} = P(A)$

③ Continuous uniform distribution

$S = [a, b]$

$P([x, y]) = \boxed{\frac{y - x}{b - a}}$

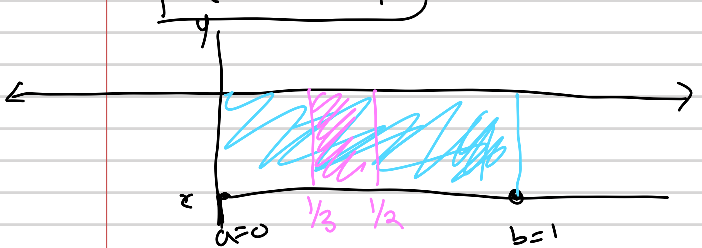




$P([x,y]) = \frac{(y-x)\cancel{h}}{(b-a)\cancel{h}} = \frac{y-x}{b-a}$

Standard uniform distribution

$\boxed{a = 0 \quad b = 1}$



$f(x) = h$

$(b-a)h = (1-0)h = 1 \cdot h = h$

$A = [\frac{1}{3}, \frac{1}{2}]$

$(\frac{1}{2} - \frac{1}{3})h = \frac{1}{6}h$

$\frac{\frac{1}{6}h}{h} = \boxed{\frac{1}{6}}$

(5) Suppose we have the following

How much wood would a woodchuck chuck if a word chuck could chuck wood?

① What is the distribution of the length of the words?

Sample space

$|S'|=6 \rightarrow S' = \boxed{\{1, 2, 3, 4, 5, 9\}}$ Must have

$|S|=9 \rightarrow S = \{1, 2, 3, 4, 5, \boxed{6, 7, 8}, 9\}$ everything else is "bonus"

If S' this the distribution?

$P(1) = \frac{2}{14}$ $P(2) = \frac{1}{14}$ $P(3) = \frac{1}{14}$ $P(4) = \frac{4}{14}$ $\neq \boxed{\frac{|A|}{|S|}}$

$P(5) = \frac{5}{14}$ $P(9) = \frac{1}{14}$

$\boxed{P(6) = P(7) = P(8) = 0}$

if S is the sample, this is the distribution

$|S''|=10$ $S'' = S' \cup \{car, fig, 77{,}327, \pi\}$

& $P(car) = P(fig) = P(77{,}327) = P(\pi) = 0$