

#4 Suppose we have a box w/ n tickets. $1-n$ tickets

$$\Omega = \{ (i, j) \mid i, j \in \{1, 2, \dots, n\} \}$$

$$= \{ (\underline{1}, \underline{1}), (\underline{1}, \underline{2}), (\underline{1}, \underline{3}), \dots, (\underline{1}, \underline{n}), \dots, (\underline{n}, \underline{1}), (\underline{n}, \underline{2}), (\underline{n}, \underline{3}), \dots, (\underline{n}, \underline{n}) \}$$

$$= \{ (\underline{i}, \underline{j}) \text{ such that } \underline{i}, \underline{j} \in \{1, 2, \dots, n\} \}$$

$$|\Omega| = n \cdot n = n^2$$

① same number twice $\Rightarrow i \& j$ are equal

$$A = \{ (i, i) \mid i \in \{1, \dots, n\} \} = \{ (1, 1), (2, 2), (3, 3), \dots, (n, n) \} \subseteq \Omega$$

$$|A| = n \Rightarrow P(A) = \frac{|A|}{|\Omega|} = \frac{n}{n^2} = \frac{1}{n}$$

② A : the two numbers are cons integers

$$\rightarrow (1, 2), (2, 5), \dots, (7, 8) \checkmark$$

$$\rightarrow (2, 1), (3, 7), \dots, (1, 2)$$

$$A = \{ (i, i+1) \mid i \in \{1, 2, \dots, n-1\} \} \cup \{ (i, i-1) \mid i \in \{2, \dots, n\} \}$$

$$\cancel{(n, n+1)}$$

$$(1, 0)$$

$$|A| = (n-1) + (n-1) = 2n-2 = 2(n-1)$$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{2n-2}{n^2} = \frac{2(n-1)}{n^2}$$

③ $\#$ pulled $\leq 2^{\text{nd}}$ $\#$ pulled

$$A = \{ (i, j) \mid i < j, i \in \{1, \dots, n-1\}, j \in \{2, \dots, n\} \}$$

$$i=1 \quad A_1 = \{ (1, j) \mid j \in \{2, \dots, n\} \} \quad |A_1| = n-1$$

$$i=2 \quad A_2 = \{ (2, j) \mid j \in \{3, \dots, n\} \} \quad |A_2| = n-2$$

$$|A| = |A_1| + |A_2| + \dots + |A_{n-1}| = \sum_{i=1}^{n-1} i = \frac{(n-1)n}{2}$$

$$n - (n-1) = n - n + 1 = 1$$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{\frac{n^2-n}{2}}{n^2} = \frac{n(n-1)}{2n^2} = \frac{n-1}{2n}$$

$$n=2 \quad 3$$

$$1+2=3$$

$$\frac{2(3)}{2} = 3 \quad n(n-1)$$

Prove

$$\#10 \quad \binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

$$\begin{aligned} (n-1)-(k-1) &= n-1-k+1 \\ &= n-k \end{aligned}$$

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{((n-1)-(k-1))!(k-1)!} + \frac{(n-1)!}{((n-1)-k)!(k)!}$$

$$= \frac{(n-1)!}{(n-k)! (k-1)!} \cdot \frac{k}{k} + \frac{(n-1)!}{(n-k-1)! k!} \cdot \frac{n-k}{n-k}$$

$$(n-k)! = (n-k)(n-k-1) \dots (2)(1) \\ = (n-k)(n-k-1)!$$

$$= \frac{k(n-1)!}{(n-k)!(k-1)!} + \frac{(n-k)(n-1)!}{(n-k)!(k)!} \leftarrow \text{Denominator } \smile$$

$$= \frac{k(n-1)! + (n-k)(n-1)!}{(n-k)! k!}$$

$$= \frac{k(n-1)! + n(n-1)! - k(n-1)!}{(n-k)! k!}$$

$$= \frac{n(n-1)!}{(n-k)! k!}$$

$$= \frac{n!}{(n-k)! k!}$$

$$= \binom{n}{k}$$

$$\frac{(n-1)!}{(n-k-1)! k!} \cdot \frac{(n-k)}{(n-k)} \quad \text{---} (n-k)$$

#1 Given a 52 card deck.

(1) How many different 5 card hands are there.

① 5 cards
Order doesn't matter
Start w/ 52 card.

choosing 5 of 52

$$\frac{n!}{k!(n-k)!} = \binom{n}{k} = \binom{52}{5} = 2,598,960 = \frac{52!}{5!47!}$$

$$= \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

(2) How many ^{5 card hands} do not have #2?

4

$$\binom{4}{1} = 4 = \frac{4!}{1!3!}$$

$$\binom{4}{3} = 4 = \frac{4!}{3!1!}$$

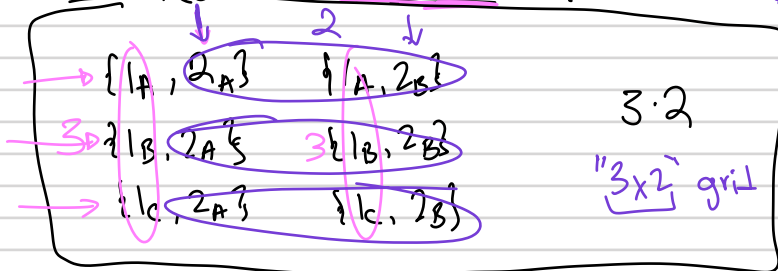
$$\binom{52-4}{5} = \binom{48}{5} = \frac{48!}{5!43!} = 1,712,302$$

(3) How many 5 card hands have exactly one two.

I want (4 cards that are not 2) and (1 card that is a 2)

$$\binom{48}{4} \cdot \binom{4}{1}$$

pull 2 cards 1 card that is not 2 & 1 that is 2.



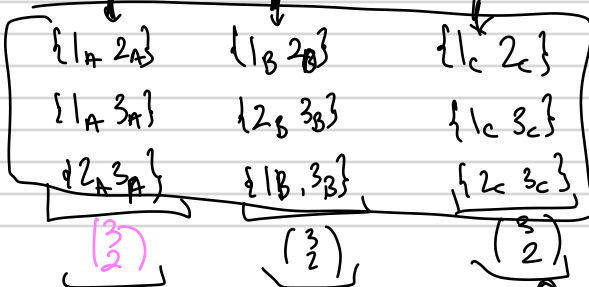
Given a 5 card hand
how many cards are
of the same suit?

$$\binom{48}{4} \cdot \binom{4}{1} = \frac{4!}{1!3!} = 4$$

(4) How many 5 card hands are of the same suit?

Smaller example: 1A 1B 1C 2A 2B 2C 3A 3B 3C

choose 2



rows

card/suit

hand

$$4 \cdot \binom{13}{5} = 5,148$$

How many
2 card hands are
there of the same
suit in the deck?

"Counting"

↳ There is no easy way.

"Combinatorics"

Combinations, permutations, orderings, & sequences.

① Try a smaller example & write all options

② Which of my tools helps me determine # of things I have.

③ Trial & Error until something works

(12)

Suppose it's your birthday & there's cake!

(7)

"there exists"

your slice is the size of both your friend's slices.

(∀) - for all (for every)

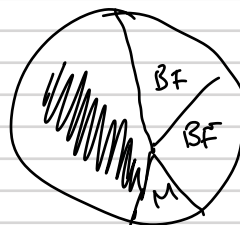
friend slice size = $S \rightarrow 2g$

your slice size = $2S (= S + S) \rightarrow 4g$

mom's slice size = $\frac{S}{2} \rightarrow g$

$$|A| = \underbrace{g}_{\text{mom}} + \underbrace{2g}_{\text{BF1}} + \underbrace{2g}_{\text{BF2}} + \underbrace{4g}_{\text{your slice}} = 9g$$

Size of cake



$$\frac{|A|}{|A|} = 4g$$

your slice

$$\frac{|A|}{|A|} = \frac{4g}{9g} = \boxed{\frac{4}{9}}$$