Indicator f^ns

X_i, I_i, 1_i ← indicator f^n

G ← is a probality #

Let I_i be the indicator f^n for the i^{th} event → Y_i

100 probabilities → $p_1, p_2, \dots, p_{100}$

$p_3, p_4, \dots, p_{100} = ?$

$\sum p_i = 1$

→ $1 = p_1 + p_2 + \dots + p_{100}$

$1 - p_1 - p_2 = p_3 + \dots + p_{100}$

Indicator function of an event

X (1 event) → will happen → p (prob.)
→ will not happen → 1-p

$I_X = \begin{cases} 1 & \text{if X happens} \\ 0 & \text{if X does not happen} \end{cases}$ ← k

~~k successes in n trials~~

Bernoulli dist. is the Binomial dist. w/ n=1

then $E(X) = p(1) + (1-p)0$

Binomial → average (mean) ↔ Expected Value

$np = \mu$

$E(X_1 + \dots + X_n) = E(X_1) + \dots + E(X_n)$ (each $E(X_i) = p$)
$= np$

→ Roll a 6-sided die → discrete uniform dist.

$E(X) = \sum x P(X=x) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \dots + 6 \cdot \frac{1}{6} = 3.5$

1 2 3 ¦ 4 5 6
3.5

Not same !!

→ What are chances of rolling a 3? Bernoulli dist.

$1 \cdot \frac{1}{6} + 0 \cdot \frac{5}{6} = \frac{1}{6}$ (indicator)

Exercise 55 Suppose I'm looking at the grades for the Monday update and the mastery quiz in class (of 250 people). For the Monday update 10% of the students got a 0 and 90% got a 1. For the mastery quiz 20% of the students got a 0 and 80% got a 1. I want to see how well my students are doing in two different ways. First I create one list where I add each student's grade from the two quizzes. On another list I multiply each student's grade from the two quizzes.

(1) Can I calculate the average grade for students on the first list? (the one I added them together) If so, what's the average?

(2) Can I calculate the average grade for students on the second list? (the one I multiplied them together) If so, what's the average?

$$E(X) = \sum_{\text{all } x} x\, P(X=x)$$

Discrete uniform distribution, average is Expected value.

$$\frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$x_1 \cdot \frac{1}{n} + x_2 \cdot \frac{1}{n} + \cdots + x_n \cdot \frac{1}{n}$$

probability

person1: u_1, q_1

person2: u_2, q_2

(1) list one: (u_1+q_1) ← 1st term, (u_2+q_2) ← 2nd term

can I calculate?

$$\frac{(u_1+q_1)+(u_2+q_2)}{2} = \frac{(u_1+u_2)}{2} + \frac{(q_1+q_2)}{2}$$

$E(U) = 0.90 \cdot 1 + 0.1 \cdot 0 = 0.9$

$$\frac{u_1+u_2+u_3+\cdots+u_{250}}{250} = 0.9$$

$E(Q) = 0.8 \cdot 1 + 0.2 \cdot 0 = 0.8$

$$\frac{q_1+q_2+\cdots+q_{250}}{250} = 0.8$$

mini-Example

$$\frac{u_1+u_2}{2} = 0.9$$

$$\frac{q_1+q_2}{2} = 0.8$$

for n:

$$\frac{(u_1+q_1)+(u_2+q_2)+\cdots+(u_{250}+q_{250})}{250}$$

$$= \frac{(u_1+u_2+\cdots+u_{250})}{250} + \frac{(q_1+q_2+\cdots+q_{250})}{250}$$

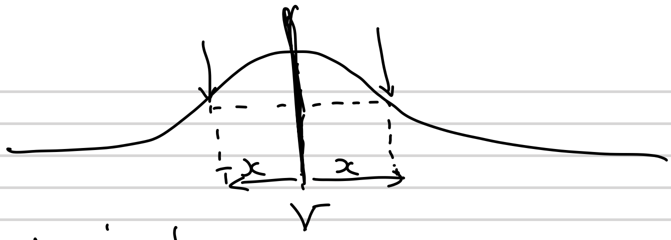
$= 0.9 + 0.8$

$= 1.7$

(2) $\frac{u_1 q_1 + u_2 q_2}{2}$ uncalculable

$(u_1+u_2)(q_1+q_2) = u_1q_1 + u_2q_2 + u_1q_2 + u_2q_1$?

Symmetry



$$P(X = r + x) = P(X = r - x) \quad \forall x$$

← if r is the middle

Ex 24.1: 101 #(0, ..., 9)

$P(5.5 - 5.5) = P(0) = \frac{1}{10}$

$P(5.5 + 5.5) = P(11) = 0$

is 3.5 the middle?

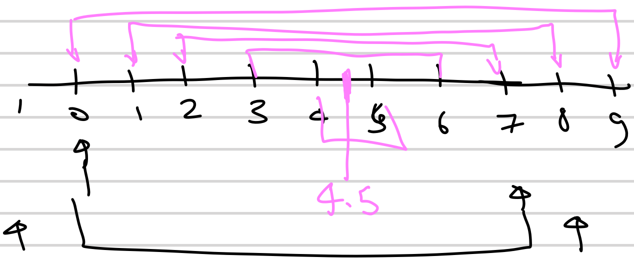
$P(3.5 + 3.5) = P(7) = \frac{1}{10}$

$P(3.5 - 3.5) = P(0) = \frac{1}{10}$

~~$P(3.5 + 4.5) = P(8) = 1/10$~~

~~$P(3.5 - 4.5) = P(-1) = 0$~~

$\neq$



$909 - 0$

$\frac{909 - 0}{2} = 454.5$

$101 \cdot 4.5 = 454.5$

is 4.5 the middle?

$P(4.5 + 5.5) = P(10) = 0$

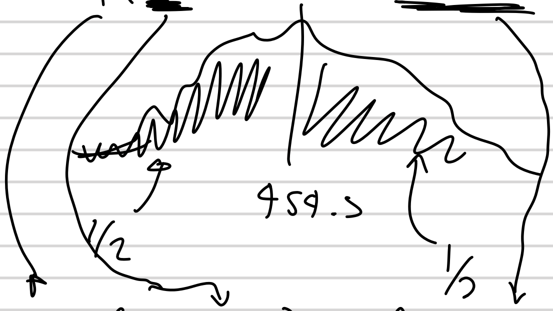
$P(4.5 - 5.5) = P(-1) = 0$

$P(4.5 + 1) = P(4.5) = 0$

$P(4.5 - 1) = P(3.5) = 0$

Any x

$P(S \leq 454.5) = P(S \geq 454.5)$



$P(S \leq 454) = P(S \geq 455)$

$P(S \leq 454) + P(S \geq 455) = 1$

$\Rightarrow P(S \leq 454) = 1/2$

Exercise 53 Let X and Y be two independent random variables which are each uniformly distributed on $\{1, 2, \dots, n\}$. Find:

$\to P(X = x, Y = y)$ (and) ind. $P(X=x)P(Y=y)$ $\to P(X=i) = \frac{1}{n}$

(1) $P(X = Y)$

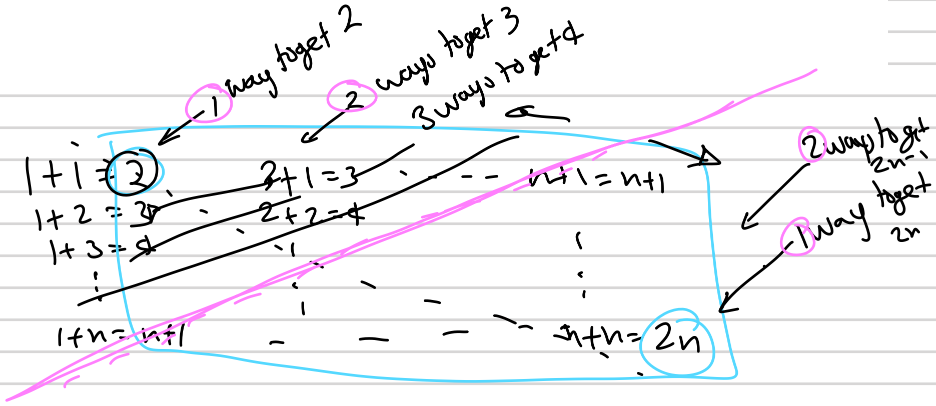
(2) $P(X < Y)$ and $P(X > Y)$

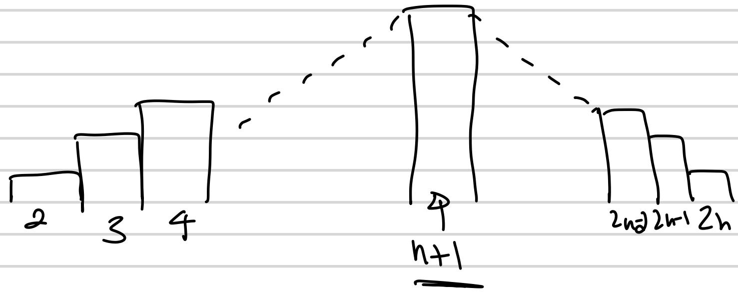
(3) $P(\max(X, Y) = k)$ for $1 \le k \le n$

(4) $P(\min(X, Y) = k)$ for $1 \le k \le n$

(5) $P(X + Y = k)$ for $2 \le k \le 2n$

$P(X+Y=k)$





$\frac{2n+2}{2} = 2\frac{(n+1)}{2} = n+1.$

$2 \le k \le n+1$

$\to P(X+Y=k) = \sum_{i=1}^{k-1} P(X=i)P(Y=k-i)$ (ind.)

$= \sum_{i=1}^{k-1} \frac{1}{n} \cdot \frac{1}{n}$

$= k-1 \cdot \frac{1}{n} \cdot \frac{1}{n} = \boxed{\frac{k-1}{n^2}}$

$n+1 < k \le 2n$

$P(X+Y=k) = P(X+Y = n+1 + (k-(n+1)))$ } symmetric about $n+1$

$= P(X+Y = n+1 - (k-(n+1)))$

$= P(X+Y = n+1-k+n+1)$

$= P(X+Y) = 2(n+1)-k)$

$= \frac{2(n+1)-k-1}{n^2}$

$P(X+Y = n+1+x)$

$= P(X+Y = n+1-x)$

why? $k > n+1$

$2(n+1)-k$

$\le 2(n+1)-(n+1)$

$\le n+1$

$\therefore$ we can use 1st part.

Solutⁿ

$$P(X+Y=k) = \begin{cases} \frac{k-1}{n^2} & \text{if } 2 \le k \le n+1 \\ \frac{2(n+1)-k-1}{n^2} & \text{if } n+1 < k \le 2n \end{cases}$$

Exercise 52 Suppose that we have m different (disjoint) random variables X_i, where each has probability p_i of occurring. Let n be the number of attempts made and let k_i be the number of times that the ith outcome happened out of the n attempts. For $i < j$:

(1) What is the distribution of X_i?

(2) What is the distribution of $X_i + X_j$?

(3) What is the joint distribution of X_i, X_j, and $n - X_i - X_j$?

(Just give a formula for all parts.)