Exercise 33 A human is playing darts and has super good aim. They're able to hit the red bullseye with probability 0.7. Assume each throw is independent and that he makes 8 throws in total.

(1) Given that they hit the bullseye at least twice, what is the chance that they hit the bullseye exactly four times?

(2) Given that the first two throws hit the bullseye, what is the chance that they hit the bullseye exactly four times in the 8 throws?

(1) P(hit bullseye 4 times | hit bullseye at least twice)

$$= \frac{P(\text{hit bullseye 4 times} \cap \text{hit bullseye at least twice})}{P(\text{hit bullseye at least twice})}$$

hitting at least 2 times = hitting 2x ⊔ hitting 3x ⊔ hitting 4x ⊔ .... ⊔ hitting 8x

hitting 4 times ⊆ hitting at least 2 times.

$$= \frac{P(\text{hit bullseyes 4 times})}{P(\text{hit bullseye at least twice})}$$

1 − P(no hits) − P(one hit)

$$= \frac{\binom{8}{4}(0.7)^4(0.3)^4}{1-\left(\binom{8}{0}(0.7)^0(0.3)^8\right)-\left(\binom{8}{1}(0.7)^1(0.3)^7\right)}$$

= 0.1363126...

Bin. dist.

$$\boxed{\binom{n}{k} p^k (1-p)^{n-k}}$$

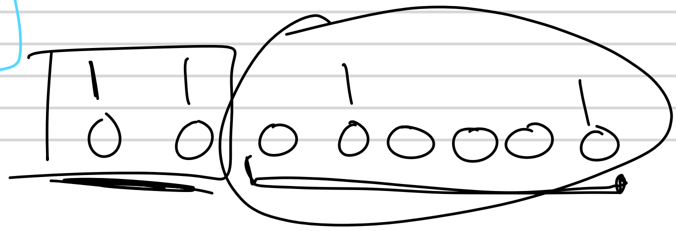
(2) P(exactly 4 hits | 2 hits in first 2 throws)

= P(2 hits in 6 throws)

k, n

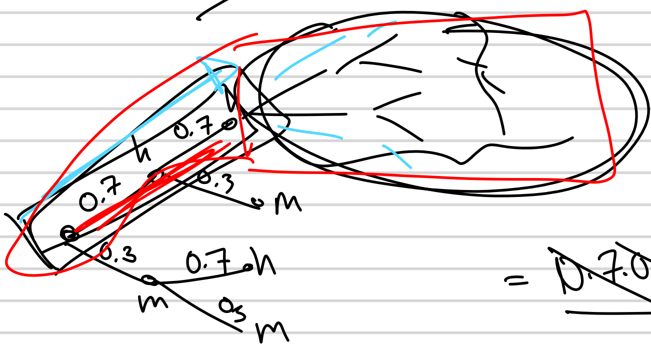
$$\boxed{= \binom{6}{2}(0.7)^2(0.3)^4}$$

$= \boxed{0.059535}$



$$P(4 \text{ hit in } 8 \mid 2 \text{ hit in first } 2)$$

$$= \frac{P(\{4 \text{ hits in } 8 \cap 2 \text{ hits in first two}\})}{P(2 \text{ hits in first two})\}\ 0.7 \cdot 0.7}$$



$$= \frac{\cancel{0.7 \cdot 0.7}\ \boxed{P(2 \text{ hit in } 6)}}{\cancel{0.7 \cdot 0.7}}$$

$$P(2 \text{ hits in a game of } 6 \mid 2 \text{ hits in another game of } 2)$$

$$= \frac{P(2 \text{ hit in a game of } 6 \cap 2 \text{ hit in another game of } 2)}{P(2 \text{ hits in another of game } 2)}$$

$$= \frac{\boxed{P(2 \text{ hits is a game of } 6)} \cdot \cancel{P(2 \text{ hit in another game of } 2)}}{\cancel{P(2 \text{ hits in another of game } 2)}}$$

Exercise 34 A battle for the royal dice thrower is about to begin. Two humans are sat in front of one another with a fair eight sided die in front of them. Each round, the two humans roll the die. If one person scores higher than another, they win the round; if the score is even, it's a tie. There are five rounds in this tournament. What are the chances that the first player wins at least four out of five rounds?

① Chance of winning a round?

(i, j)

player 1 player 2

$$P(i>j) \stackrel{?}{=} \frac{28}{64} = \boxed{\frac{7}{16}} = p$$

$$\boxed{\binom{n}{k}(p)^k(1-p)^{n-k}}$$

$(8, j) \Rightarrow j \in \{1, 2, \dots, 7\}$ 7

$(7, j) \Rightarrow j \in \{1, 2, \dots, 6\}$ + 6

+ 5 + 4 + 3 + 2 + 1 = 28

②
$$\underbrace{\binom{5}{4}\left(\frac{7}{16}\right)^4\left(\frac{9}{16}\right)^1}_{\text{exactly } 4} + \underbrace{\binom{5}{5}\left(\frac{7}{16}\right)^5\left(\frac{9}{16}\right)^0}_{\text{exactly } 5}$$

$$\frac{9}{16} = 1 - \frac{7}{16}$$

$\frac{7}{16} \to p$, $1-p$

$$= \boxed{0.119068}$$

$$\boxed{\binom{5}{4}} = \frac{5!}{4!(5-4)!} = \frac{5\cancel{!}}{\cancel{4!} \cdot 1!} = \frac{5}{1!} = 5$$

$\boxed{{}_nC_r \quad {}_nC_k}$ → calculator or program

→ Never use these written.