"$f(x) = x^2$"

Binomial vs. Normal Dist (& Approximat$^{\underline{n}}$)

Binomial: (independent) event → successful w/ prob. p. — Bernoulli distribution

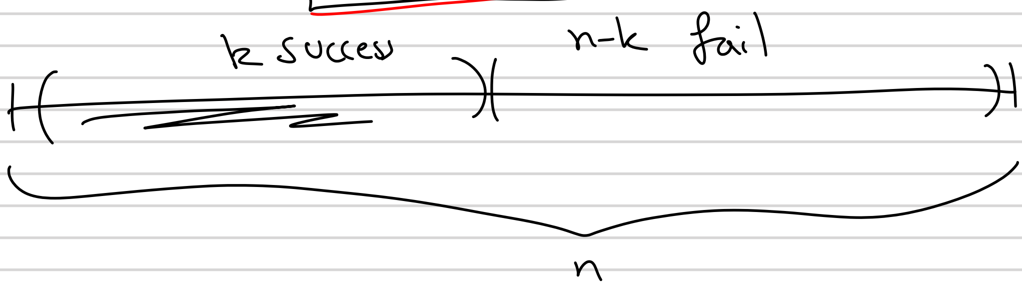
$p(\text{success}) = p$

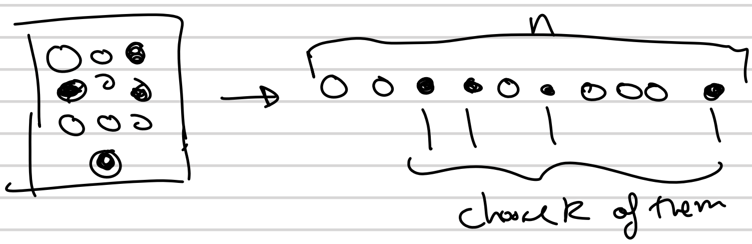
$p(\text{fail}) = 1-p$

→ do this event n.

$P(\text{all success}) = p^n$

$P(k \text{ success}) = \boxed{\binom{n}{k} p^k (1-p)^{n-k}}$ — binomial distribut$^{\underline{n}}$ → "$g(x) = (f(x))^n$"



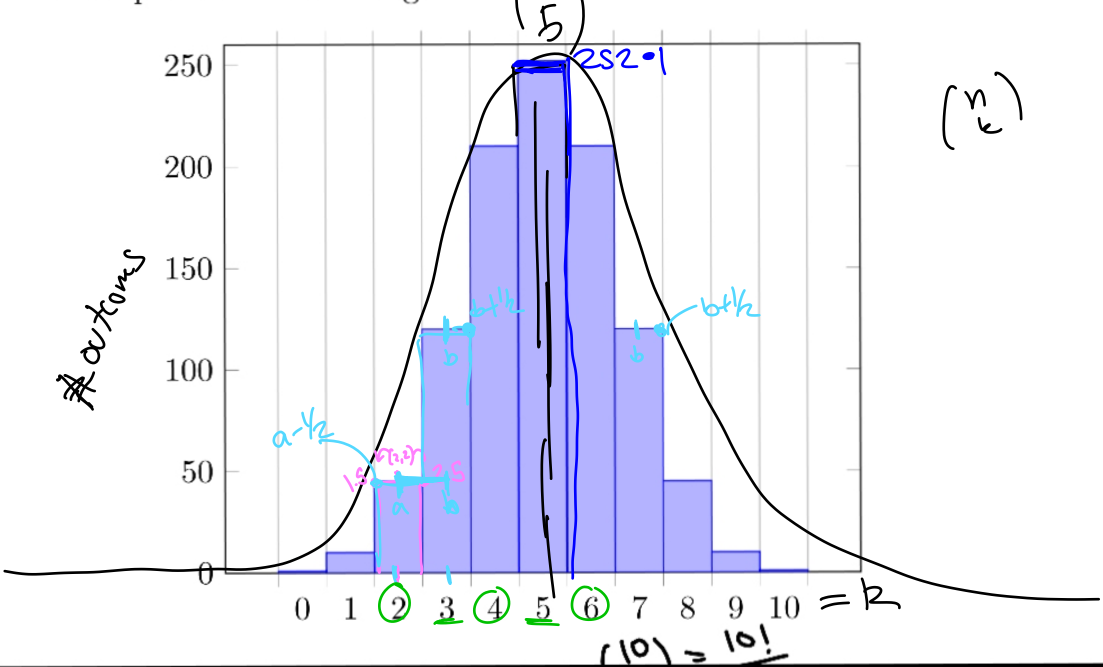


distribution: P a function $p(A) \geq 0$

- $p(A) = p(A_1) + P(A_2) + \cdots + P(A_n)$ (partition)
- $p(\Omega) = 1$

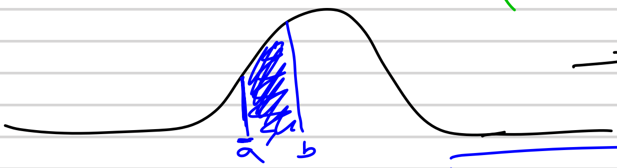
Example 12.1 Let's look at the distribution when $p = \frac{1}{2}$ and $n = 10$.

We end up with the following:



$P(2) + P(4) + P(6)$
$\approx \left(P(2+\tfrac{1}{2}\ldots) - P(2-\tfrac{1}{2}\ldots)\right) + (\quad) + (\quad)$

Normal Distribution:



$\rightarrow$ is the f^n $\boxed{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}}$

normal distribution

$P(\text{a to b successes}) = \boxed{\int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} dx} = \int_a^b \Phi'\left(\frac{x-\mu}{\sigma}\right) dx$

$P(\Omega) = 1 \Rightarrow P(-\infty \text{ to } \infty \text{ successes}) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} dx = 1$

$\boxed{\Phi\left(\frac{b-\mu}{\sigma}\right)} - \Phi\left(\frac{a-\mu}{\sigma}\right)$

Expected val: $\mu = np$

Standard dev $\sigma = \sqrt{n(p)(1-p)}$

$P(\text{a to b successes}) \approx P(\text{a to b successes}) =$

binomial distribution

normal distribution

(better) $\approx \Phi\left(\frac{b+\frac{1}{2}-\mu}{\sigma}\right) - \Phi\left(\frac{a-\frac{1}{2}-\mu}{\sigma}\right)$

(37) Life savings = \$25

bet 25 times $(n=25)$

win = +2 → ≥ \$25, I need to win at least 13 $(13 \cdot 2 = 26)$

lose = +0

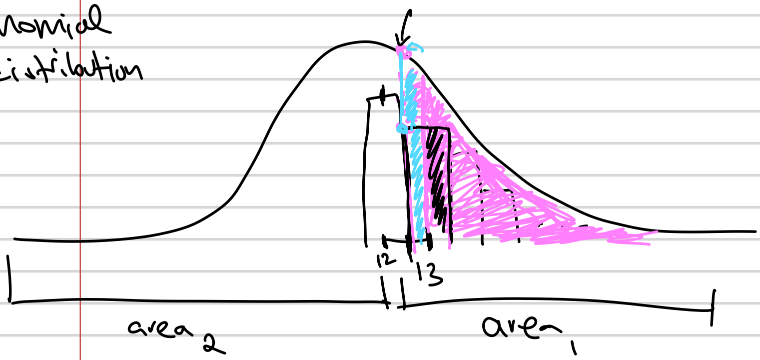
$P(\text{"get a red"}) = \frac{18}{38} = p$

verb

Bernoulli distribution

$P(\text{" win \$25 or more "}) = P(\text{win} \geq 13 \text{ times}) = 1 - \Phi\left(\frac{13 - \frac{1}{2} - \mu}{\sigma}\right)$

Binomial distribution



$\text{area}_1 = 1 - \text{area}_2$

Bernoulli = chance that 1 event happens = the chance that I get a red

$= P(\text{"get a red"})$

p

Bernoulli

$\mu = np = 25 \cdot \frac{18}{38} = 11.84$

$\sigma = \sqrt{np(1-p)} = \sqrt{\mu(1-p)} = \sqrt{11.84\left(1 - \frac{18}{38}\right)} = 2.5$

$P(\text{"win} \geq 13 \text{ times"})$

$\parallel$

$$P(\geq 13) = 1 - \Phi\left(\frac{13 - \frac{1}{2} - \mu}{\sigma}\right) = 1 - \Phi\left(\frac{13 - \frac{1}{2} - 11.84}{2.5}\right)$$

$$= 1 - \Phi(0.264)$$

$$= 1 - 0.6026$$

$$= 0.3974$$

(38) 0.45 enjoyed $(p=0.45)$

200 people responded $(n=200)$

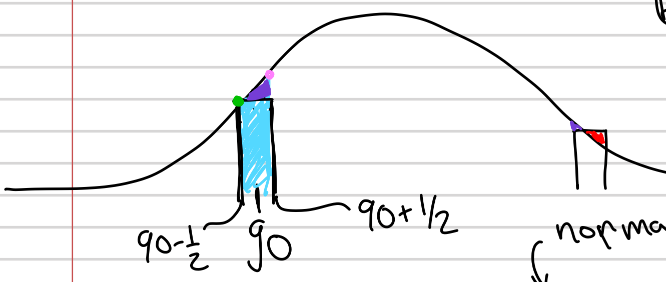
$\mu = np = 200 \cdot 0.45 = 90$

$\sigma = \sqrt{np(1-p)} = \sqrt{90(0.55)} = 7.035$

(1) chance exactly 90 people enjoyed prod out of 200

$$\underset{\text{Binomial}}{P}(90 \text{ enjoyed}) = \underbrace{\binom{\overset{n}{200}}{\underset{k}{90}} (\underset{p}{0.45})^{\overset{k}{90}} (\underset{1-p}{0.55})^{\overset{n-k}{110}}}$$

$$\frac{200!}{110!\,90!}$$



normal approximat^{n}

$$\underset{\text{Binomial}}{P}(90 \text{ enjoyed}) \approx \Phi\left(\frac{90+\frac{1}{2}-\mu}{\sigma}\right) - \Phi\left(\frac{90-\frac{1}{2}-\mu}{\sigma}\right)$$

$$= \Phi\left(\frac{90+\frac{1}{2}-90}{7.035}\right) - \Phi\left(\frac{90-\frac{1}{2}-90}{7.035}\right)$$

$$= \Phi\left(\frac{1}{2 \cdot 7.035}\right) - \Phi\left(-\frac{1}{2 \cdot 7.035}\right)$$

$\Phi(0.071) = 0.5279$

(0.4721)

$$= 0.5279 - (1 - 0.5279)$$

$$= \underline{0.0558}$$

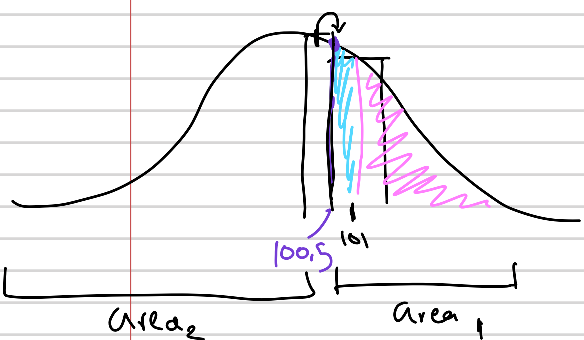
$(= 0.05653)$

(2) over 50% enjoyed product

200 people, 50% of 200 is 100.

P(over 100 enjoyed) = P(at least 101 enjoyed)

$\underline{P(>100)}$ $\qquad$ $P(\geq 101)$



$$P(\geq 101) \approx 1 - \Phi\left(\frac{101-\frac{1}{2}-\mu}{\sigma}\right)$$

$$= 1 - \Phi\left(\frac{101-\frac{1}{2}-90}{7.035}\right)$$

$$= 1 - \Phi(1.49) = 1 - 0.9319 = \boxed{0.0681}$$

$$P(\geq 101) = \sum_{k=101}^{200} \binom{200}{k} (0.45)^{k} (0.55)^{200-k}$$