Week 7

Exercise 67   Suppose we have a coin which lands on heads with probability p. Suppose that we toss the coin a lot of times and that each toss is independent.

 (1) What is the probability that the fifth head appears on the 12th toss?

 (2) What is the probability that the same number of heads appear in the first eight tosses as in the next five tosses?

Exercise 68   Suppose that we have two independent geometric random variables X_1 and X_2. Let X_1 have parameter p_1 and X_2 have parameter p_2. Find the following:

 (1) $P(X_1 = X_2)$

 (2) $P(X_1 < X_2)$

(For the second one, you can leave it in a summation.)

Exercise 69   Suppose you're playing volleyball where the first team to get two points more than their opponents is declared the winner. Let G be the total number of games you played where each game is independent of all other games. Assume that your team ends up winning each game with probability p.

 (1) What is the probability that the total number of games played is n?

 (2) Find the expected value of G.

 (3) Find the variance of G.

Exercise 70   Suppose we're making chocolate chip cookies and we want the probability of a cookie containing at least 1 chocolate chip to be at least 99%. How many chocolate chips must a cookie contain on average?

Exercise 71   You've been hired by Aram to make sure his exercises don't have errors. On average there is 1 error per page. What is the probability that after Aram has written all 300 pages of his exercise sheets there will be at least one page that has at least 5 errors on it? (Assume Poisson distribution and that each page having errors is independent.)

Exercise 72   Suppose it's a snowy day and roughly 30 snowflakes are falling per square inch per minute. What is the chance that any particular square inch is *not* hit by any snowflakes during a given 10 second period? (Use Poisson distribution)

Exercise 73   We're on a class field trip to Chernobyl! In order to be safe, we brought a Geiger counter to make sure we're not going to die from radiation. We notice that we receive roughly 10 pulses per minute. What is the probability that we receive 3 pulses in any given half-minute period? (Use Poisson distribution)

Exercise 74   Instead of Chernobyl, pretend we're in a laboratory where we're looking into radioactive substances (safely). We know that radioactive substances release particles known as α-particles. We set-up a counter (like a Geiger counter) to see how many α-particles are given off in a time period. Suppose that we have two different substances and they are emitting α-particles independently of one another. The first substance has a Poisson distribution with parameter 3.87 and the second substance has a Poisson distribution with parameter 5.41 (in a given time frame). What is the probability that the counter is hit by at most 4 α-particles (based on the time frames given by the distributions)?

Exercise 75   Suppose that X has density function $f(x) = cx^2(1-x)^2$ for $0 < x < 1$ and is equal to 0 everywhere else.

 (1) What is the value of c?

 (2) What is the expected value of X?

 (3) What is the variance of X?