Week 6

Exercise 57 Suppose that you go and buy some scratchers to test your luck. Of the $2,000$ scratchers, 250 of them are winners and the rest lose. You decide to buy 5 scratchers. (Scratchers are those lottery things that you have to scratch off to see if you win or not)

(1) What is the expected number of winning scratchers you will buy?

(2) What is an upper bound for the probability of having at least one winning scratcher?

(3) What is the exact probability of having at least one winning scratcher?

Exercise 58 Suppose you're waiting for the elevator in the Burj Khalifa. There are 163 floors! You walk into the elevator and 200 little kids rush into the elevator after you. (They have special elevators like in *Doctor who* that are bigger on the inside). Before you can get out, the doors close and you notice each kid start randomly (independently) choosing a random floor! You're now stuck, so you decide to just enjoy the view as the elevator goes up and stops a bunch of times. How many times do you expect the elevator to stop in total?

Exercise 59 Suppose that I look at a standard deck of 52 cards which are randomly and fairly shuffled. Let X_1 be the number of cards which appear before the first ace. Let X_2 be the number of cards which appear before the second ace. Let X_3 and X_4 be the number of cards which appear before the third and fourth aces respectively. Let X_5 be the number of cards which appear after the last ace. Suppose that each X_i has the same distribution.

(1) What is the probability $P(X_i = k)$ for some value $0 \leq k \leq 48$.

(2) What is the expected value of each X_i? (Hint, don't use the probability)

(3) Are the X_i pairwise independent?

Exercise 60 Difficult problem alert! Suppose that X is a random variable with potential values 0, 1 and 2. What is the distribution of X based on the expected values $E(X) = \mu$ and $E(X^2) = \xi$?

Exercise 61 Suppose that X represents the number of heads obtained if a (normal, fair) coin is tossed three times. Find the expected value and the variance of X^2.

Exercise 62 Suppose that we have three different events: A, B, and C. Let $P(A) = \frac{1}{5}$, $P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{3}$. Let X be the total number of events A, B and C that occur.

(1) What is the expected value of X?

(2) If the events are disjoint, what is the variance?

(3) If the events are independent, what is the variance?

Exercise 63 In Toronto, the average income is roughly $\$65,000$. According to a report by Fong and Partners, you need an annual salary of $\$135,000$ to be considered "middle class".

(1) Find an upper bound for the percentage of individuals who are considered at least middle class, *i.e.*, their income is over $135,000$.

(2) According to statcan, the standard of deviation of incomes for $25 - 29$ year olds, is $\$30,000$. Can you find a better upper bound for the percentage of individuals (aged $25 - 29$) who are considered at least middle class?

(Hint: you need to use two different inequalities)

Exercise 64 According to the book, in roulette, a "house special" is a bet on the numbers 0, 00, 1, 2 and 3 out of the 38 total possible numbers. So you have a $\frac{5}{38}$ chance of winning and for every dollar you bet, you get seven dollars back if you win (and nothing if you lose). If you make the "house special" bet 300 times where you bet \$1 each time, what is the chance that you have at least as much money as you started with? (In other words, what's the probability you end with greater than or equal to \$300.) Use normal approximation.

Exercise 65 Suppose you were walking down the street and you found \$100,000 just lying there with no one around. You pick it up and decide to keep it because, we're in a word problem so why not? You then decide to invest that money like a responsible adult. There are four different stock options, but you don't know which will give you a profit and which will take your money. You know one of them gives you a profit of 200 dollars for every 1000 you invest, another one gives 100 for every 1000 you invest, a third gives you nothing for every 1000 you invest and the last one makes you lose 100 for every 1000 you invest.

(1) Suppose you randomly choose to invest all $100,000$ into one stock. What is the probability that you will earn (at least) $8,000$ dollars?

(2) Suppose you decide to invest 1000 dollars, 100 times where each time you randomly chose a stock to invest in. What is the probability that you will earn (at least) $8,000$ dollars? (Use the central limit theorem)

Exercise 66 Let D_i be a random digit between 0 and 9. Let X_i be the last digit of D_i^2 (so if $D_i = 8$ then $D_i^2 = 64$ and $X_i = 4$). Let $\bar{X}_n = \frac{X_1+...+X_n}{n}$ be the average of a large number n where each X_i is determined from randomly chosen D_i.

(1) Predict the value of $\bar{X}_n$ for large n.

(2) If you just had to predict the last digit of $\bar{X}_{100}$, what digit should you chose to maximize your chance of being right, and what is the probability?