Week 5

Exercise 52 Suppose that we have m different (disjoint) random variables X_i, where each has probability p_i of occurring. Let n be the number of attempts made and let k_i be the number of times that the ith outcome happened out of the n attempts. For $i < j$:

(1) What is the distribution of X_i?

(2) What is the distribution of $X_i + X_j$?

(3) What is the joint distribution of X_i, X_j, and $n - X_i - X_j$?

(Just give a formula for all parts.)

Exercise 53 Let X and Y be two independent random variables which are each uniformly distributed on $\{1, 2, \ldots, n\}$. Find:

(1) $P(X = Y)$

(2) $P(X < Y)$ and $P(X > Y)$

(3) $P(\max(X, Y) = k)$ for $1 \leq k \leq n$

(4) $P(\min(X, Y) = k)$ for $1 \leq k \leq n$

(5) $P(X + Y = k)$ for $2 \leq k \leq 2n$

Exercise 54 Suppose that you throw three (fair six-sided) dice and look at the sum. What are the chances that the sum of the numbers is 11 or more? (You should be able to do this without a calculator.)

Exercise 55 Suppose I'm looking at the grades for the Monday update and the mastery quiz in class (of 250 people). For the Monday update 10% of the students got a 0 and 90% got a 1. For the mastery quiz 20% of the students got a 0 and 80% got a 1. I want to see how well my students are doing in two different ways. First I create one list where I add each student's grade from the two quizzes. On another list I multiply each student's grade from the two quizzes.

(1) Can I calculate the average grade for students on the first list? (the one I added them together) If so, what's the average?

(2) Can I calculate the average grade for students on the second list? (the one I multiplied them together) If so, what's the average?

Exercise 56 What is the expected value of the number of spades drawn if you pull 7 cards randomly from a (well-shuffled, normal) deck of 52 cards?