

Week 4

Exercise 41 In order to get more money, an airline uses the fact that only 90% of people who book a flight, show up at the airport. On a particular flight with 300 seats, the airline decided to book 324 people.

- (1) Assuming independence, what is the chance that the flight will be overbooked? (More than 300 people will show up.)
- (2) Assuming people travel in pairs, what is the chance that the flight will be overbooked? (Assume the probability of showing up doesn't change)

(Use normal approximations and then redo with skew normal approximations.)

Exercise 42 Suppose that we roll a fair die 100 times. We know that the probability of getting 25 or more rolls of six happens with probability 0.022. If you rolled the fair die 100 times every day for a year (365 days), find the chance that you have 25 or more sixes:

- (1) At least once
- (2) At least twice

(Use Poisson approximation)

Exercise 43 A hat contains 1000 random numbers in it. Only two of the numbers are prime and the rest are composite (not prime).

- (1) What is more likely to occur if we pull 1000 numbers from the box with replacement:
 - Less than 2 prime numbers are pulled
 - Exactly 2 prime numbers are pulled
 - More than 2 prime numbers are pulled
- (2) If we repeat the 1000 pulls just like the previous question, but this time do it twice in a row (so, do 1000 pulls with replacement; and then do a second 1000 pulls with replacement); what are the chances that we get the same number of prime numbers in both 1000 pulls?

(Use Poisson approximation)

Exercise 44 Suppose you have a normal deck of 52 cards and you deal out 3 cards randomly. Recall that there are 26 red cards and 26 black cards.

- (1) What's the probability that the first card is red and the second two are black?
- (2) What's the probability that exactly one card is red?
- (3) What's the probability that at least one card is red?

Exercise 45 In the magical forest, out of every 100,000 creatures, 40% are unicorns and 60% are horses. Say we pull a random sample of 100 creatures without replacement. What's the exact probability that there are at least 45 unicorns in the sample? (Don't solve. Keep it as a summation) Using normal approximation, give an approximation of the probability instead.

Exercise 46 Suppose you go to a small fair with 9 of your friends and you each purchase 10 tickets in the fair's raffle. Later on, you found out, your group of 10 people were the only people to purchase tickets for the raffle! Out of the 100 tickets bought, 3 of them will be chosen randomly as winning tickets.

- (1) What are the chances one person will get all 3 winning tickets?
- (2) What are the chances that 3 different people will win something?
- (3) What are the chances that 2 different people will win?

Exercise 47 Let X be the number of tails in three random tosses of a fair coin. Write the distribution of X in a table and find the distribution of $|X - 1|$.

Exercise 48 Let X and Y be the numbers obtained in two pullings of tickets out of a hat which has four tickets labelled 1, 2, 3, 4. Find the joint distribution table for X and Y if we do the sampling with and without replacement. In each of the cases find the probability $P(X \leq Y)$.

Exercise 49 Let X_1 and X_2 be the numbers obtained by rolling two rolls of a fair die. Let $Y_1 = \max(X_1, X_2)$ and $Y_2 = \min(X_1, X_2)$. We already calculated the joint distribution table for X_1 and X_2 in class. What is the joint distribution table for Y_1 and Y_2 ?

Exercise 50 Let X be the number of heads which appear in 20 random tosses of a fair coin. Let Y be a number which is randomly picked from a hat with the numbers $\{0, 1, 2, \dots, 20\}$ (independently from X). Let $Z = \max(X, Y)$. Find a formula for $P(Z = k)$ where $0 \leq k \leq 20$.

Exercise 51 Suppose that we have two dice which are not fair. The number i appears with probability p_i for the first die and with probability p'_i for the second die. Let S be the sum of numbers rolled with these two dice. Find the formula for $P(S = 2)$, $P(S = 7)$ and $P(S = 12)$.