# Week 11

# 16–20 Mar 2020

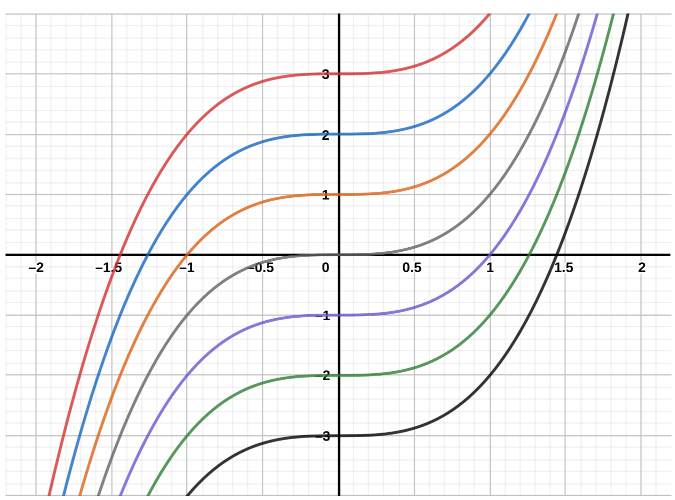
## 11.1 Antiderivatives - §4.9

We've been looking at how to find the derivative of a function $f(x)$. But what happens if we want to go backwards? Say we have some function $f(x)$ and we want to know whose derviative it is. This is the idea of antiderivatives.

**Definition 11.1** A function $F$ is called an *antideriative* of $f$ on an interval $I$ if ____________________ for all $x$ in $I$.

**Example 11.2** If $f(x) = 3x^2$, what is its antiderivative? __________

What do these look like?



But how do we know that's all? What if there is some other anti-derivative? This turns out not to be the case because of the following theorem.

**Theorem 11.3** *If $F$ is an antiderivative of $f$ on an interval $I$, then the most general antiderivative of $f$ on $I$ is*

**Example 11.4** What are the (most general) antiderivatives of the following functions:

(1) $f(x) = \cos(x)$ : ________________

(2) $f(x) = x^n$ where $n \neq 1$ : ________________

(3) $f(x) = x^{-1}$ : ________________

We can use our derivative laws to help us find the antiderivatives of more complicated functions. For example, if we have $f(x) + g(x)$ and if $F(x)$ and $G(x)$ are the antiderivatives of $f$ and $g$ respectively, then $F(x) + G(x)$ is an antiderivative of $f(x) + g(x)$. Let's see an example.

**Example 11.5** Find all antiderivatives of $f(x) = \frac{1}{\sqrt{1-x^2}} - 2\cos(x) + \frac{2x^5 + \sqrt[3]{x}}{x^2}$

**Exercise 11.6** Try one with a partner. Let $f(x) = 2\sin(x) - \frac{1+x^3}{x}$ and find all antiderivatives of $f(x)$.

If we're looking for a particular answer, we also give a data point with our function.

**Example 11.7** Find $f$ if $f'(x) = e^x - 2\cos(x)$ and $f(0) = 2$.

Let's try something a little harder.

**Example 11.8** Find $f$ if $f''(x) = 3x + 2$ where $f(0) = 4$ and $f(2) = 13$.

**Exercise 11.9** With a partner, find $f$ if $f''(x) = 4x^2 + 1$ where $f(0) = \frac{1}{6}$ and $f(1) = 2$.

| Function | Derivative | Function | Derivative |
|---|---|---|---|
| $f(x) = x^r$ | $f'(x) = rx^{r-1}$ | $f(x) = kg(x)$ | $f'(x) = kg'(x)$ |
| $f(x) = g(x)h(x)$ | | $f(x) = \frac{g(x)}{h(x)}$ | $f'(x) = \frac{g'(x)h(x)-g(x)h'(x)}{h^2(x)}$ |
| $f(x) = g(h(x))$ | | $f(x) = g(x)^r$ | $f'(x) = r(g(x))^{r-1}g'(x)$ |
| $f(x) = e^x$ | $f'(x) = e^x$ | $f(x) = \ln(x)$ | $f'(x) = \frac{1}{x}$ |
| $f(x) = a^x$ | $f'(x) = a^x \ln(a)$ | $f(x) = \log_a(x)$ | $f'(x) = \frac{1}{x\ln(a)}$ |
| $f(x) = \sin(x)$ | | $f(x) = \cos(x)$ | |
| $f(x) = \tan(x)$ | $f'(x) = \sec^2(x)$ | $f(x) = \cot(x)$ | $f'(x) = -\csc^2(x)$ |
| $f(x) = \sec(x)$ | $f'(x) = \sec(x)\tan(x)$ | $f(x) = \csc(x)$ | $f'(x) = -\csc(x)\cot(x)$ |
| $f(x) = \arcsin(x)$ | $f'(x) = \frac{1}{\sqrt{1-x^2}}$ | $f(x) = \arccos(x)$ | $f'(x) = \frac{-1}{\sqrt{1-x^2}}$ |
| $f(x) = \arctan(x)$ | $f'(x) = \frac{1}{1+x^2}$ | $f(x) = \text{arccot}(x)$ | $f'(x) = \frac{-1}{\sqrt{1+x^2}}$ |
| $f(x) = \text{arcsec}(x)$ | $f'(x) = \frac{1}{\|x\|\sqrt{x^2-1}}$ | $f(x) = \text{arccsc}(x)$ | $f'(x) = \frac{-1}{\|x\|\sqrt{x^2-1}}$ |