Homework 5 solutions

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3 February 2020

An exercise marked with the symbol ⋆ is considered more difficult and will not be an exam question.

Exercise 1 Differentiate the following functions

(1) $f(x) = e^5$

(2) $g(x) = \frac{7}{4}x^2 - 3x + 12$

(3) $f(t) = 1.4t^5 - 2.5t^2 + 6.7$

(4) $H(u) = (3u - 1)(u + 2)$

(5) $B(y) = y^-6$

(6) $y = x^{5/3} - x^{2/3}$

(7) $h(t) = \sqrt[4]{t} - 4e^t$

(8) $y = \sqrt[3]{x}(2 + x)$

(9) $S(R) = 4\pi R^2$

(10) $y = \frac{\sqrt{x}+x}{x^2}$

(11) $G(t) = \sqrt{5}t + \frac{\sqrt{7}}{t}$

(12) $k(r) = e^r + r^e$

(13) $F(z) = \frac{A+Bz+Cz^2}{z^2}$

(14) $D(t) = \frac{1+16t^2}{(4t)^3}$

(15) $y = e^{x+1} + 1$

Solution. (1) $f'(x) = 0$

(2) $g'(x) = \frac{7}{2}x - 3$

(3) $f'(t) = 7t^4 - 5t$

(4) $H'(u) = 6u + 5$

(5) $B'(y) = -6y^{-7}$

(6) $y' = \frac{5}{3}x^{2/3} - \frac{2}{3}x^{-1/3}$

(7) $h'(t) = \frac{1}{4}t^{-3/4} - 4e^t$

(8) $y' = \frac{2}{3}x^{-2/3} + \frac{4}{3}x^{1/3}$

(9) $S'(R) = 8\pi R$

(10) $y' = \frac{-3}{2}x^{-5/2} - x^{-2}$

(11) $G'(t) = \frac{\sqrt{5}}{2\sqrt{t}} - \frac{\sqrt{7}}{t^2}$

(12) $k'(r) = e^r + er^{e-1}$

(13) $F'(z) = -\frac{2A+Bz}{z^3}$

(14) $D'(t) = -\frac{3}{64t^4} - \frac{1}{4t^2}$

(15) $y' = e^{x+1}$

□

Exercise 2 Suppose we are looking at a population growth where the growth rate by year is given by $f(t) = 3e^t + 7t$. What is the instantaneous rate of growth at years 1, 2 and 3?

Solution. The formula for instantaneous rate of growth is $f'(t) = 3e^t + 7$. Therefore at years 1, 2 and 3 we have:

$$f'(1) = 3e + 7 \approx 15.155 \quad f'(2) = 3e^2 + 7 \approx 29.167 \quad f'(3) = 3e^3 + 7 \approx 67.257$$

□

Exercise 3 Differentiate the following functions:

(1) $g(x) = (x + 2\sqrt{x})e^x$

(2) $y = \frac{e^x}{1-e^x}$

(3) $G(x) = \frac{x^2-2}{2x+1}$

(4) $J(v) = (v^3 - 2v)(v^{-4} + v^{-2})$

(5) $f(z) = (1 - e^z)(z + e^z)$

(6) $y = \frac{\sqrt{x}}{2+x}$

(7) $y = \frac{1}{t^3+2t^2-1}$

(8) $h(r) = \frac{ae^3}{b+e^r}$

(9) $y = (z^2 + e^z)\sqrt{z}$

(10) $V(t) = \frac{4+t}{te^2}$

(11) $F(t) = \frac{At}{Bt^2+Ct^3}$

(12) $f(x) = \frac{ax+b}{cx+d}$

Solution. (1) $g'(x) = e^x \left(x + 2\sqrt{x} + 1 + \frac{1}{\sqrt{x}}\right)$

(2) $y' = \frac{e^x}{(1-e^x)^2}$

(3) $G'(x) = \frac{2x^2+2x+4}{(2x+1)^2}$

(4) $J'(v) = 1 + v^{-2} + 6v^{-4}$

(5) $f'(z) = 1 - ze^z - 2e^{2z}$

(6) $y' = \frac{2-x}{2\sqrt{x}(2+x)^2}$

(7) $y' = -\frac{3t^2+4t}{(t^3+2t^2-1)^2}$

(8) $h'(r) = \frac{abe^r}{(b+e^r)^2}$

(9) $y' = \frac{5z^2+e^z+2ze^z}{2\sqrt{z}}$

(10) $V'(t) = -\frac{(t+2)^2}{t^2e^t}$

(11) $F'(t) = -\frac{A(B+2Ct)}{t^2(B+Ct)^2}$

(12) $f'(x) = \frac{ad-bc}{(cx+d)^2}$

□

Exercise 4 Find the first and second derivatives of the following functions:

(1) $f(x) = \sqrt{x}e^x$

(2) $f(x) = \frac{x}{x^2-1}$

Solution. (1) (a) $f'(x) = \left(x^{1/2} + \frac{1}{2}x^{-1/2}\right)e^x$

(b) $f''(x) = \frac{4x^2+4x-1}{4x^{3/2}}e^x$

(2) (a) $f'(x) = \frac{-x^2-1}{(x^2-1)^2}$

(b) $f''(x) = \frac{2x^3+6x}{(x^2-1)^3}$

□

Exercise 5 Suppose that we are using Ohm's law to calculate the instantaneous voltage at a given time measured in minutes. Suppose that the current at any time is given by $i(t) = 3t^2 + t$ and the resistance at any time is given by $r(t) = e^t$. What is the instantaneous voltage at 1, 2 and 3 minutes from the start?

Solution. Recall that the instantaneous voltage is given by:

$$i'(t)r(t) + r'(t)i(t)$$

Since $i'(t) = 6t + 1$ and $r'(t) = e^t$, therefore:

$$v(t) = (6t + 1)e^t + e^t(3t^2 + t) = (3t^2 + 7t + 1)e^t$$

Therefore we have:

$$v(1) = 11e \approx 29.901 \quad v(2) = 27e^2 \approx 199.505 \quad v(3) = 49e^3 \approx 984.191$$

$\square$