Homework 10

by Aram Dermenjian

16 March 2020

An exercise marked with the symbol $\star$ is considered more difficult and will not be an exam question.

Exercise 1 Find the most general antiderivative of the following functions.

(1) $f(x) = x^2 - 3x + 2$

(2) $f(x) = 6x^5 - 8x^4 - 9x^2$

(3) $f(x) = (x-5)^2$

(4) $f(x) = x^{3.4} - 2x^{\sqrt{2}-1}$

(5) $f(x) = e^2$

(6) $f(x) = \sqrt[3]{x^2} + x\sqrt{x}$

(7) $\star f(t) = \frac{3t^4 - t^3 + 6t^2}{t^4}$

(8) $r(\theta) = \sec(\theta)\tan(\theta) - 2e^\theta$

(9) $g(v) = 2\cos(v) - \frac{3}{\sqrt{1-v^2}}$

(10) $f(x) = 1 + 2\sin(x) + \frac{3}{\sqrt{x}}$

(11) $f(x) = \frac{2x^2+5}{x^2+1}$

Solution. We let c be a constant.

(1) $\frac{x^3}{3} - \frac{3}{2}x^2 + 2x + c$

(2) $x^6 - \frac{8}{5}x^5 - 3x^3 + c$

(3) $\frac{1}{3}x^3 - 5x^2 + 25x + c$

(4) $\frac{x^{4.4}}{3.4} - \sqrt{2}x^{\sqrt{2}} + c$

(5) $e^2 x + c$

(6) $\frac{3x^{5/3}}{5} + \frac{2x^{5/2}}{5} + c$

(7) When $t < 0$ then $3t - \ln(|t|) - 6/t + c_1$ and when $t > 0$ then $3t - \ln(t) - 6/t + c_2$.

(8) $\sec(\theta) - 2e^\theta + c_n$ on the interval $(n\pi - \pi/2, n\pi + \pi/2)$.

(9) $2\sin(v) - 3\arcsin(v) + c$

(10) $x - 2\cos(x) + 6\sqrt{x} + c$

(11) $2x + 3\arctan(x) + c$

□

Exercise 2 Find f.

(1) $f''(x) = x^6 - 4x^4 + x + 1$

(2) $f''(x) = \frac{1}{x^2}$

(3) $f'''(t) = \sqrt{t} - 2\cos(t)$

(4) $f'(x) = 5x^4 - 3x^2 + 4$ where $f(-1) = 2$

(5) $f'(t) = t + \frac{1}{t^3}$ where $t > 0$ and $f(1) = 6$

(6) $f'(x) = \frac{x+1}{\sqrt{x}}$ where $f(1) = 5$

(7) $f'(t) = 3^t - 3/t$ where $f(1) = 2$ and $f(-1) = 1$

(8) $f''(x) = 8x^3 + 5$ where $f(1) = 0$ and $f'(1) = 8$

(9) $f''(t) = t^2 + \frac{1}{t^2}$ where $t > 0$ and $f(2) = 3$ and $f'(1) = 2$

(10) $f''(t) = \sqrt[3]{t} - \cos(t)$ where $f(0) = 2$ and $f(1) = 2$

(11) $f'''(x) = \cos(x)$ where $f(0) = 1$, $f'(0) = 2$ and $f''(0) = 3$

Solution. Let c and d be constants.

(1) $f(x) = \frac{x^8}{56} - \frac{2x^6}{15} + \frac{x^3}{6} + \frac{x^2}{2} + cx + d$

(2) $f(x) = -\ln(-x) + c_1x + d_1$ if $x < 0$ and $-\ln(x) + c_2x + d_2$ when $x > 0$.

(3) $f(t) = \frac{8t^{7/2}}{105} + 2\sin(t) + ct^2 + dt + e$

(4) $f(x) = x^5 - x^3 + 4x + 6$

(5) $f(t) = \frac{t^2}{2} - \frac{1}{2t^2} + 6$

(6) $f(x) = \frac{2x^{3/2}}{3} + 2\sqrt{x} + \frac{7}{3}$

(7) $f(t) = \frac{3^t}{\ln(3)} - 3\ln(-t) + 1 - \frac{1}{3\ln(3)}$ when $t < 0$ and $\frac{3^t}{\ln(3)} - 3\ln(t) + 2 - \frac{3}{\ln(3)}$ when $t > 0$.

(8) $f(x) = \frac{2x^5}{5} + \frac{5x^2}{2} + x - \frac{39}{10}$

(9) $f(t) = \frac{t^4}{12} - \ln(t) + \frac{8}{3}t + \ln(2) - \frac{11}{3}$

(10) $f(t) = \frac{9t^{7/3}}{28} + \cos(t) + \left(\frac{19}{28} - \cos(1)\right)t + 1$

(11) $f(x) = -\sin(x) = \frac{3x^2}{2} + 3x + 1$

□