Homework 9

by Aram Dermenjian

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An exercise marked with the symbol $\star$ is considered more difficult and will not be an exam question.

Exercise 1 For the following:

- Find the intervals on which f is increasing/decreasing.
- Find the local max/min values of f.
- Find the intervals of concavity and the inflection points.

(1) $f(x) = 2x^3 - 9x^2 + 12x - 3$

(2) $f(x) = \frac{x}{x^2+1}$

(3) $f(x) = \cos^2(x) - 2\sin(x)$, $0 \leq x \leq 2\pi$.

(4) $f(x) = x^2 \ln(x)$

(5) $f(x) = x^4 e^{-x}$

Exercise 2 Sketch the graph of a function that satisfies all the conditions.

(1)
- Vertical assymptote at $x = 0$.
- $f'(x) > 0$ if $x < -2$ and $f'(x) < 0$ if $x > -2$.
- $f''(x) < 0$ if $x < 0$ and $f''(x) > 0$ if $x > 0$.

(2)
- $f'(0) = f'(4) = 0$
- $f'(x) = 1$ if $x < -1$
- $f'(x) > 0$ if $0 < x < 2$
- $f'(x) < 0$ if $-1 < x < 0$ or $2 < x < 4$ or $x > 4$
- $\lim_{x\to 2^-} f(x) = \infty$ and $\lim_{x\to 2^+} f(x) = \infty$.
- $f''(x) > 0$ if $-1 < x < 2$ or $2 < x < 4$
- $f''(x) < 0$ if $x > 4$.

Exercise 3 Find the limit using L'hôpital's rule where appropriate. If you aren't allowed to use L'hôpital's rule, explain why.

(1) $\lim_{x\to 3} \frac{x-3}{x^2-9}$

(2) $\lim_{x\to -2} \frac{x^3+8}{x+2}$

(3) $\lim_{x\to 1/2} \frac{6x^2+5x-4}{4x^2+16x-9}$

(4) $\lim_{x\to 0} \frac{\tan(3x)}{\sin(2x)}$

(5) $\lim_{x\to 0} \frac{x^2}{1-\cos(x)}$

(6) $\lim_{\theta\to \pi} \frac{1+\cos(\theta)}{1-\cos(\theta)}$

(7) $\lim_{x\to \infty} \frac{x+x^2}{1-2x^2}$

(8) $\lim_{x\to \infty} \frac{\ln(\sqrt{x})}{x^2}$

(9) $\lim_{t\to 0} \frac{8^t-5^t}{t}$

(10) $\lim_{u\to \infty} \frac{e^{u/10}}{u^3}$

(11) $\lim_{x\to 0} \frac{x-\sin(x)}{x-\tan(x)}$

(12) $\lim_{x\to \infty} \frac{(\ln(x))^2}{x}$

(13) $\lim_{x\to 0} \frac{\cos(mx)-\cos(nx)}{x^2}$

(14) $\lim_{x\to 1} \frac{x\sin(x-1)}{2x^2-x-1}$

(15) $\star\lim_{x\to 0^+} \frac{x^x-1}{\ln(x)+x-1}$

(16) $\lim_{x\to 0} \frac{e^x-e^{-x}-2x}{x-\sin(x)}$

(17) $\lim_{x\to \infty} \sqrt{x}e^{-x/2}$

(18) $\lim_{x\to -\infty} x\ln\left(1-\frac{1}{x}\right)$

(19) $\lim_{x\to \infty} x^{3/2}\sin(1/x)$

(20) $\lim_{x\to ((\pi/2)^-)} \cos(x)\sec(5x)$

(21) $\lim_{x\to 0} (\csc(x)-\cot(x))$

(22) $\lim_{x\to 0^+} \left(\frac{1}{x}-\frac{1}{\arctan(x)}\right)$

(23) $\lim_{x\to 1^+} \left(\ln(x^7-1)-\ln(x^5-1)\right)$

(24) $\star\lim_{x\to 0^+} \left(\tan(2x)\right)^x$

(25) $\lim_{x\to \infty} x^{(\ln(2))/(1+\ln(x))}$

(26) $\lim_{x\to \infty} x^{e^{-x}}$

(27) $\lim_{x\to 1}(2-x)^{\tan(\pi x/2)}$

(28) $\lim_{x\to \infty} \left(\frac{2x-3}{2x+5}\right)^{2x+1}$

Exercise 4 Find two numbers whose difference is 100 and whose product is a minimum.

Exercise 5 The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

Exercise 6 Find the dimensions of a rectangle with area 1000 m^2 whose perimeter is as small as possible.

Exercise 7 A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs 10 per square meter. Material for the sides cost 6 per square meter. Find the cost of materials for the cheapest such container.