# Homework 8

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An exercise marked with the symbol $\star$ is considered more difficult and will not be an exam question.

**Exercise 1** Find the differential of each function.

(1) $y = \frac{1+2u}{1+3u}$

(2) $y = \theta^2 \sin(2\theta)$

(3) $y = \ln(\sin(\theta))$

(4) $y = \frac{e^x}{1-e^x}$

**Exercise 2** Sketch the graph of $f$ by hand and use your sketch to find the absolute/local max and min values of $f$.

(1) $f(x) = 2 - \frac{x}{3}$ when $x \geq -2$

(2) $f(x) = \frac{1}{x}$ when $1 < x < 3$

(3) $f(x) = \sin(x)$ when $0, < x \leq \pi/2$

(4) $f(x) = \cos(t)$ when $-3\pi/2 \leq t \leq 3\pi/2$

(5) $f(x) = |x|$

(6) $f(x) = e^x$

**Exercise 3** Find the critical numbers of the following functions.

(1) $f(x) = x^3 + 6x^2 - 15x$

(2) $f(x) = 2x^3 + x^2 + 2x$

(3) $g(t) = |3t - 4|$

(4) $h(p) = \frac{p-1}{p^2+4}$

(5) $g(x) = \sqrt[3]{4 - x^2}$

(6) $g(\theta) = 4\theta - \tan(\theta)$

(7) $h(t) = 3t - \arcsin(t)$

(8) $f(x) = x^{-2} \ln(x)$

**Exercise 4** Find the abs. max and abs. min values of the following $f$ on the given intervals.

(1) $f(x) = 5 + 54x - 2x^3$, $[0, 4]$

(2) $f(x) = x^3 - 6x^2 + 5$, $[-3, 5]$

(3) $f(t) = (t^2 - 4)^3$, $[-2, 3]$

(4) $f(t) = \frac{\sqrt{t}}{1+t^2}$, $[0, 2]$

(5) $f(t) = t + \cot(t/2)$, $[\pi/4, 7\pi/4]$

(6) $f(x) = xe^{x/2}$, $[-3, 1]$

(7) $f(x) = x - 2\tan^{-1}(x)$, $[0, 4]$

**Exercise 5** Show that $x^3 + e^x = 0$ has exactly one real root.

**Exercise 6** Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers $c$ that satisfy the conclusion of the MVT.

(1) $f(x) = x^3 - 3x + 2$, $[-2, 2]$

(2) $f(x) = 1/x$, $[1, 3]$