Homework 3

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An exercise marked with the symbol $\star$ is considered more difficult and will not be an exam question.

Exercise 1 Prove the following limit using an ε, δ proof.

$$\lim_{x\to 0} x^3 = 0$$

Exercise 2 Prove the following limit using an ε, δ proof.

$$\lim_{x\to -6^+} \sqrt[8]{6+x} = 0$$

Exercise 3 Evaluate the following limits and justify each step.

(1)
$$\lim_{x\to -1}(x^4-3x)(x^2+5x+3)$$

(2)
$$\lim_{u\to -2}\sqrt{u^4+3u+6}$$

(3)
$$\lim_{t\to 2}\left(\frac{t^2-2}{t^3-3t+5}\right)^2$$

Exercise 4 Evaluate the following limits, if they exist.

(1)
$$\lim_{x\to -3}\frac{x^2+3x}{x^2-x-12}$$

(2)
$$\lim_{x\to 4}\frac{x^2+3x}{x^2-x-12}$$

(3)
$$\lim_{x\to -1}\frac{2x^2+3x+1}{x^2-2x-3}$$

(4)
$$\lim_{h\to 0}\frac{(2+h)^3-8}{h}$$

(5)
$$\lim_{t\to 1}\frac{t^4-1}{t^3-1}$$

(6)
$$\lim_{u\to 2}\frac{\sqrt{4u+1}-3}{u-2}$$

(7)
$$\lim_{h\to 0}\frac{(3+h)^{-1}-3^{-1}}{h}$$

(8)
$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)$$

(9)
$$\lim_{x\to 2}\frac{x^2-4x+4}{x^2-3x^2-4}$$

(10)
$$\lim_{x\to -4}\frac{\sqrt{x^2+9}-5}{x+4}$$

(11)
$$\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-1\frac{1}{x^2}}{h}$$

Exercise 5 Use the squeeze theorem to show

$$\lim_{x\to 0}\sqrt{x^3+x^2}\sin\left(\frac{\pi}{x}\right)=0$$

Exercise 6 If $2x \le g(x) \le x^4 - x^2 + 2$, for all x, find $\lim_{x\to 1} g(x)$.

Exercise 7 Find the following limits, if they exist.

(1)
$$\lim_{x\to -6}\frac{2x+12}{|x+6|}$$

(2)
$$\lim_{x\to -2}\frac{2-|x|}{2+x}$$

(3)
$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{|x|}\right)$$

Exercise 8 Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

(1) $g(t)=\frac{t^2+5t}{2t+1}$, $a=2$

(2) $f(x) = 3x^4 - 5x + \sqrt[3]{x^2+4}$, $a = 2$

Exercise 9 Explain why the functions below iare discontinous at the given number a.

(1) for $a = -2$,

$$f(x) = \begin{cases} \frac{1}{x+2} & \text{if } x \neq -2 \\ 2^x & \text{if } x = 2 \end{cases}$$

(2) for $a = -1$,

$$f(x) = \begin{cases} \frac{x^2-x}{x^2-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$$

(3) for $a = 3$,

$$f(x) = \begin{cases} \frac{2x^2-5x-3}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$

Exercise 10 Explain using our theorems, why the functions below are continuous everywhere on its domain. What are their domains?

(1) $G(x) = \frac{x^2+1}{2x^2-x-1}$

(2) $R(t) = \frac{e^{\sin(t)}}{2+\cos(\pi t)}$

(3) $B(x) = \frac{\tan(x)}{\sqrt{4-x^2}}$

(4) $N(r) = \tan^{-1}(1 + e^{-r^2})$

Exercise 11 Locate the dicontinuities of $y = \ln(\tan^2(x))$.

Exercise 12 Use the intermediate value theorem to show that there is a root of the given equation in the specified interval.

(1) $\ln(x) = x - \sqrt{x}$ on $(2, 3)$.

(2) $\sin(x) = x^2 - x$ on $(1, 2)$.