Homework 11

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23 March 2020

An exercise marked with the symbol $\star$ is considered more difficult and will not be an exam question.

Exercise 1 State the terms of the following sums.

(1) $\sum_{j=2}^{5} 4\,j^3 - 1$

(2) $\sum_{k=1}^{4} (-2)^k - \frac{1}{2^k}$

(3) $\sum_{i=2}^{7} (-3)^{i-4}$

(4) $\sum_{i=1}^{3} f(x_i)^i$

Exercise 2 Use the $\sum$ symbol to represent the following sums.

(1) $2 + 4 + 6 + 8$

(2) $-\frac{1}{2} + \frac{1}{4} + -\frac{1}{6} + \frac{1}{8} + -\frac{1}{10}$

(3)$\star$ $-1 + 2 + -3 + 4 + -5 + 6 + -7$

(4) $f(\frac{x_1}{1}) - f(\frac{x_2}{2}) + f(\frac{x_3}{3}) - f(\frac{x_4}{4})$

Exercise 3 Evaluate the following sums using the formulas from class.

(1) $\sum_{i=5}^{75} i$

(2) $\sum_{i=1}^{20} \frac{3}{2}\,i - \frac{5}{2}$

(3) $\sum_{i=1}^{25} (2\,i - 3)^2$

(4)$\star$ $\sum_{i=1}^{25} 5\,\left(\frac{1}{4}\right)^i + \frac{1}{2}$

(4) $\sum_{i=6}^{72} \frac{i+2}{3} - \frac{i+3}{3}$

Exercise 4 Use the formulas from class to express the following sums as functions of n.

(1) $\sum_{i=1}^{n-1} i$

(2) $\sum_{i=1}^{n-1} \frac{3\,i^2}{5\,n}$

(3) $\sum_{i=1}^{n-1} 6\,i^2 - 2\,i$

(4)$\star$ $\sum_{i=10}^{2n} i^2 - 3\,i$

Exercise 5 Show that.

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Exercise 6 ($\star$) Show that

$$\sum_{k=0}^{n-1} ar^k = a\frac{1-r^n}{1-r}$$

Exercise 7 For each of the following intervals, evaluate the length of Δx for each of the subintervals if we separate the interval into n equal parts. Give each interval.

(1) $[0, 1]$, $n = 5$

(2) $[-2, \frac{3}{2}]$, $n = 10$

(3) $[1, 5]$, $n = 12$

Exercise 8 If $f(x)$ and $g(x)$ are two integrable functions on the given intervals, and knowing that

$$\int_3^6 f(x)\,dx = 12, \qquad \int_6^{10} f(x)\,dx = 1$$
$$\int_3^0 g(x)\,dx = -1, \qquad \int_0^6 g(x)\,dx = 15$$
$$\int_6^{10} g(x)\,dx = 11$$

evaluate the following definite integrals using the properties of definite integrals from class.

(1) $\int_3^{10} f(x)\,dx$

(2) $\int_{10}^{6} f(x)\,dx$

(3) $\int_6^6 f(x)\,dx$

(4) $\int_3^{10} (3f(x) + 2g(x))\ dx$

(5) $\int_3^6 (2f(x) + 4g(x))\ dx$