Tutorial 1

Question 1 Let $\mathcal{B} = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be a basis for $\mathbb{E}^3$. Let $\mathcal{C} = (\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3)$ where

$$\begin{aligned}\mathbf{f}_1 &= \mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3 \\ \mathbf{f}_2 &= \mathbf{e}_1 + \lambda\mathbf{e}_3, \quad \lambda \in \mathbb{R} \\ \mathbf{f}_3 &= -\mathbf{e}_1 + 2\mathbf{e}_2 - \mathbf{e}_3\end{aligned}$$

(1) For what values of λ is $\mathcal{C}$ a basis?

(2) When $\mathcal{C}$ is not a basis, find a linear dependence between $\{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\}$.

Question 2 (1) Which of the following are inner products on $\mathbb{E}^2$? [where $\mathbf{x} = [x_1, x_2]^\mathsf{T}$, $\mathbf{y} = [y_1, y_2]^\mathsf{T}$ in some basis.]

$$\langle \mathbf{x}, \mathbf{y} \rangle = 0 \qquad \langle \mathbf{x}, \mathbf{y} \rangle = x_1 + y_2$$
$$\langle \mathbf{x}, \mathbf{y} \rangle = x_1x_2 + y_1y_2 \qquad \langle \mathbf{x}, \mathbf{y} \rangle = x_1y_2 + x_2y_1$$

(2) (*) Define an inner product on a basis of $\mathbb{E}^2$ by

$$\langle \mathbf{e}_1, \mathbf{e}_1 \rangle = a,\ \langle \mathbf{e}_1, \mathbf{e}_2 \rangle = \langle \mathbf{e}_2, \mathbf{e}_1 \rangle = b,\ \langle \mathbf{e}_2, \mathbf{e}_2 \rangle = c, \quad a, b, c \in \mathbb{R}$$

and extending linearly to all of $\mathbb{E}^2$. What conditions must a, b, c satisfy for this to define an inner product?

The following identity may be of use:

$$ax_1^2 + 2bx_1x_2 + cx_2^2 = \frac{(ax_1 + bx_2)^2 + (ac - b^2)x_2^2}{a}$$