Homework 12

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30 March 2020

An exercise marked with the symbol $\star$ is considered more difficult and will not be an exam question.

Exercise 1 Determine an antiderivative of the following functions.

(1) $e^{\left(\frac{3}{4}\,x\right)}$

(2) $\cos\left(\frac{1}{3}\,x\right)$

(3) $\operatorname{cosec}^2(-2x)$

(4) $x^{\frac{4}{7}}$

Solution. (1) $\frac{4}{3}\,e^{\left(\frac{3}{4}\,x\right)}$

(2) $3\,\sin\left(\frac{1}{3}\,x\right)$

(3) $-\frac{1}{2\,\tan(2\,x)}$

(4) $\frac{7}{11}\,x^{\frac{11}{7}}$

□

Exercise 2 Evaluate the following expressions:

(1) $\int_0^{27} x^{\frac{4}{3}} + \sqrt{3}\sqrt{x}\,dx$

(2) $\int_0^{\ln(3)} e^{(-4\,x)}\,dx$

(3) $\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \frac{1}{\sin(x)^2}\,dx$

Solution. (1) $\frac{7695}{7}$

(2) $\frac{20}{81}$

(3) 1

□

Exercise 3 ($\star$) Evaluate the limit by interpreting it like a definite integral on the interval $[a, b]$.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i}{n}\right)^3 \left(\frac{3}{n}\right)$$

Solution. $\frac{81}{4}$ □

Exercise 4 Evaluate the following definite integrals.

(1) $\int_{\ln(2)}^{\ln(6)} e^{(2\,x+3)}\,dx$

(2) $\int_0^2 x^2 - 2\,x + 2\,dx$

(3) $\int_1^3 \frac{8}{x}\,dx$

(4) $\int_0^{2\pi} \sin(x)\,dx$

Solution. (1) $16\,e^3$

(2) $\frac{8}{3}$

(3) $8\,\log(3)$

(4) 0

□

Exercise 5 Evaluate the following indefinite integrals.

(1) $\int \frac{\cos(x)}{\sin(x)^2}\,dx$

(2) $\int x^7\,dx$

(3) $\int x^3 + x^2 + x + 1\,dx$

(4) $\int \frac{1}{x^2+1}\,dx$

(5) $\int \cos(3\,x)\,dx$

(6) $\int dx$

(7) $\int -2\,\cos(x)^2 + \cos(2\,x)\,dx$

(8) $\int \frac{\sin(2\,\theta)}{\sin(\theta)}\,dx$

(9)⋆ $\int \frac{5}{\sin(\theta)+1}\,dx$

(10) $\int \frac{1}{\sqrt{-4\,x^2+4}}\,dx$

Solution. (1) $-\frac{1}{\sin(x)} + c$

(2) $\frac{1}{8}\,x^8 + c$

(3) $\frac{1}{4}\,x^4 + \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + x + c$

(4) $\arctan(x) + c$

(5) $\frac{1}{3}\,\sin(3\,x) + c$

(6) $x + c$

(7) $-x + c$

(8) $\frac{x\,\sin(2\,\theta)}{\sin(\theta)} + c$

(9) $\frac{5\,x}{\sin(\theta)+1} + c$

(10) $\frac{1}{2}\arcsin(x)+c$

□

Exercise 6 Evaluate

$$\int_1^2 \frac{x^3+5\,x+4}{x^3}\,dx$$

Solution. 5 □

Exercise 7 Calculate the following integrals.

(1) $\int \sqrt{2\,x+3}\,dx$

(2) $\int -4\left(3\,x^2-5\right)^5 x\,dx$

(3) $\int \left(x^3-4\right)x\,dx$

(4) $\int \sec(x)^3\tan(x)\;dx$

(5) $\int \frac{3\,x}{\sqrt{-x^2+1}}\,dx$

(6) $\int \frac{\sec\left(-\sqrt{x}+3\right)}{\sqrt{x}}\,dx$

(7) $\int \frac{1}{(4\,x-3)^2}\,dx$

(8) $\int \frac{12\,x^2}{x^3+8}\,dx$

(9) $\int \cos(2\,x)^4\sin(2\,x)\;dx$

(10) $\int \frac{1}{\sqrt{x}\left(\sqrt{x}+5\right)}\,dx$

(11) $\int \frac{1}{\sin(2\,x)^2}+\sin\left(\frac{1}{2}\,x\right)\;dx$

(12) $\int \frac{3\,x+5}{x^2+1}\,dx$

(13)⋆ $\int \frac{\log(x)^2}{x}\,dx$

(14)⋆ $\int \frac{x^2-x-1}{x-1}\,dx$

(15)⋆ $\int \frac{2\left(4\,x^2-x-3\right)}{4\,x-3}\,dx$

(16)⋆ $\int \frac{\cos(x)}{\sin(x)^3+3\,\sin(x)^2+3\,\sin(x)+1}\,dx$

Solution. (1) $\frac{1}{3}\left(2\,x+3\right)^{\frac{3}{2}}+c$

(2) $-\frac{1}{9}\left(3\,x^2-5\right)^6+c$

(3) $\frac{1}{5}\,x^5-2\,x^2+c$

(4) $\frac{1}{3\,\cos(x)^3}+c$

(5) $-3\sqrt{-x^2+1}+c$

(6) $2\log\left(\sec\left(\sqrt{x}-3\right)+\tan\left(\sqrt{x}-3\right)\right)+c$

(7) $-\frac{1}{4\,(4\,x-3)}+c$

(8) $4\log\left(x^3+8\right)+c$

(9) $-\frac{1}{10}\cos\left(2\,x\right)^5+c$

(10) $2\log\left(\sqrt{x}+5\right)+c$

(11) $-\frac{1}{2\tan(2\,x)}-2\cos\left(\frac{1}{2}\,x\right)+c$

(12) $5\arctan\left(x\right)+\frac{3}{2}\log\left(x^2+1\right)+c$

(13) $\frac{1}{3}\log\left(x\right)^3+c$

(14) $\frac{1}{2}\,x^2-\log\left(x-1\right)+c$

(15) $x^2+x-\frac{3}{4}\log\left(4\,x-3\right)+c$

(16) $-\frac{1}{2\left(\sin(x)^2+2\sin(x)+1\right)}+c$

□

Exercise 8 Calculate the following integrals.

(1) $\int_0^1 3\left(x^3+1\right)^7x^2\,dx$

(2) $\int_{1/2}^{5/4}\frac{6\,x+1}{3\,x^2+x-1}\,dx$

(3) $\int_{\frac{1}{2}}^1\frac{1}{(2\,x+1)^2}\,dx$

(4) $\int_0^{\frac{\pi}{2}}\frac{\cos(x)}{(\sin(x)+1)^2}\,dx$

(5) $\int_{-1}^0 e^{(3\,x+5)}\,dx$

(6) $\int_0^2 2\,(2\,x+1)e^{\left(x^2+x\right)}\,dx$

(7)⋆ $\int_0^1\sqrt{-4\,x^4+8\,x^2}\,dx$

Solution. (1) $\frac{255}{8}$

(2) $\log\left(\frac{79}{16}\right)+2\log\left(2\right)$

(3) $\frac{1}{12}$

(4) $\frac{1}{2}$

(5) $\frac{1}{3}\,e^5-\frac{1}{3}\,e^2$

(6) $2\,e^6-2$

(7) $-\frac{4}{3}\sqrt{2}+\frac{2}{3}$

□