Homework 6

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An exercise marked with the symbol $\star$ is considered more difficult and will not be an exam question.

Exercise 1 Differentiate the following functions.

(1) $f(x) = x\cos(x) + 2\tan(x)$

(2) $y = 2\sec(x) - \csc(x)$

(3) $g(\theta) = e^{\theta}(\tan(\theta) - \theta)$

(4) $f(t) = \frac{\cos(t)}{e^t}$

(5) $y = \sin(\theta)\cos(\theta)$

(6) $y = \frac{\cos(x)}{1-\sin(x)}$

(7) $y = \frac{\sin(t)}{1+\tan(t)}$

(8) $f(t) = te^t\cot(t)$

Exercise 2 Prove that $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$.

Exercise 3 Find the following limits.

(1) $\lim_{x\to 0}\frac{\sin(x)}{\sin(\pi x)}$

(2) $\lim_{\theta\to 0}\frac{\cos(\theta)-1}{\sin(\theta)}$

(3) $\lim_{x\to 0}\frac{\sin(3x)\sin(5x)}{x^2}$

(4) $\star$ $\lim_{x\to 0}\csc(x)\sin(\sin(x))$

(5) $\lim_{x\to 0}\frac{\sin(x^2)}{x}$

(6) $\lim_{x\to 1}\frac{\sin(x-1)}{x^2+x-2}$

Exercise 4 Find the derivative of the following functions.

(1) $F(x) = (1 + x + x^2)^{99}$

(2) $f(x) = \frac{1}{\sqrt[3]{x^2-1}}$

(3) $g(\theta) = \cos^2(\theta)$

(4) $f(t) = t\sin(\pi t)$

(5) $g(x) = e^{x^2 - x}$

(6) $g(x) = \left(x^2 + 1\right)^3 \left(x^2 + 2\right)^6$

(7) $F(t) = (3t - 1)^4 (2t + 1)^{-3}$

(8) $y = \left(x + \frac{1}{x}\right)^5$

(9) $f(t) = 2^{t^3}$

(10) $s(t) = \sqrt{\frac{1+\sin(t)}{1+\cos(t)}}$

(11) $f(z) = e^{z/(z-1)}$

(12) $J(\theta) = \tan^2(n\theta)$

(13) $F(t) = \frac{t^2}{\sqrt{t^3+1}}$

(14) $U(y) = \left(\frac{y^4+1}{y^2+1}\right)^5$

(15) $y = x^2 e^{-1/x}$

(16) $y = \sqrt{1 + xe^{-2x}}$

(17) $y = e^{\sin(2x)} + \sin(e^{2x})$

(18) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

(19) $y = 2^{3^{4^x}}$

(20) $y = \left(x + (x + \sin^2(x))^3\right)^4$

Exercise 5 Find the first and second derivatives:

(1) $y = \frac{1}{(1+\tan(x))^2}$

(2) $y = e^{e^x}$

Exercise 6 Find $\frac{dy}{dx}$ for the following by implicit differentiation:

(1) $2x^2 + xy - y^2 = 2$

(2) $x^3 - xy^2 + y^3 = 1$

(3) $xe^y = x - y$

(4) $\cos(xy) = 1 + \sin(y)$

(5) $e^y \sin(x) = x + xy$

(6) $xy = \sqrt{x^2 + y^2}$

(7) $x\sin(y) + y\sin(x) = 1$

(8) $\tan(x-y) = \frac{y}{1+x^2}$

Exercise 7 Find the following derivatives.

(1) $y = \arctan(x^2)$

(2) $g(x) = \arccos(\sqrt{x})$

(3) $y = \tan^{-1}\left(x - \sqrt{1+x^2}\right)$

(4) $R(t) = \arcsin(\frac{1}{t})$

(5) $y = \cos^{-1}\left(\sin^{-1}(t)\right)$

(6) $y = \arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$

Exercise 8 Differentiate the following functions.

(1) $f(x) = x\ln(x) - x$

(2) $f(x) = \ln(\sin^2(x))$

(3) $y = \frac{1}{\ln(x)}$

(4) $f(x) = \log_{10}(\sqrt{x})$

(5) $g(t) = \sqrt{1+\ln(t)}$

(6) $h(x) = \ln(x + \sqrt{x^2-1})$

(7) $P(v) = \frac{\ln(v)}{1-v}$

(8) $y = \ln(|1 + t - t^3|)$

(9) $y = \ln(\csc(x) - \cot(x))$

(10) $H(z) = \ln\left(\sqrt{\frac{a^2-z^2}{a^2+z^2}}\right)$

(11) $y = \log_2\left(x\log_5(x)\right)$

Exercise 9 Find y' and y'' of the follwoing functions.

(1) $y = \frac{\ln(x)}{1+\ln(x)}$

(2) $y = \ln(1+\ln(x))$

Exercise 10 Differentiate the following using the logarithm method we learned in class.

(1) $y = \frac{e^{-x}\cos^2(x)}{x^2+x+1}$

(2) $y = \sqrt{x}e^{x^2-x}(x+1)^{2/3}$

(3) $y = x^{\cos(x)}$

(4) $y = \sqrt{x}^{x}$

(5) $y = (\sin(x))^{\ln(x)}$

(6) $y = (\ln(x))^{\cos(x)}$

Exercise 11 ($\star$) Find y' if $y^x = x^y$.

Exercise 12 Find the 9th derivative of $f(x) = x^8 \ln(x)$.