Homework 1

by Aram Dermenjian

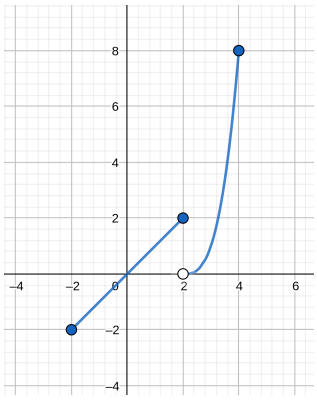
6 January 2020

An exercise marked with the symbol $\star$ is considered more difficult and will not be an exam question.

Exercise 1 Let $f(x) = x + \sqrt{4-x} + x^2$ and $g(u) = u + \sqrt{4-u} + u^2$. Is it true that $f = g$?

Exercise 2 Let $f(x) = \frac{x^2-x}{x-1}$ and $g(x) = x$. Is it true that $f = g$?

Exercise 3 Let f be the function with the following graph:



What is the domain and range of f?

Exercise 4 Give the domain and range of each of the following functions.

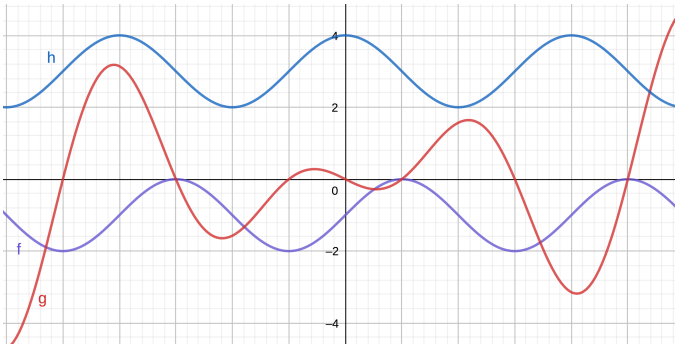
(1) $f(x) = \sqrt{x}$

(2) $f(x) = \sqrt{x+1}$

(3) $f(x) = \frac{1}{x}$

(4) $f(x) = \frac{1}{1-x^2}$

Exercise 5 Let f, g and h be the functions with the following graphs:



For each, determin whether the function is even, odd or neither.

Exercise 6 Determin whether the function f is even, odd or neither.

(1) $f(x) = \frac{x}{x^2+1}$

(2) $f(x) = \frac{x^2}{x^4+1}$

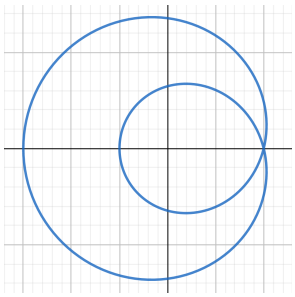
(3) $f(x) = 1 + 3x^3 - x^5$

(4) $f(x) = \frac{x}{x+1}$

Exercise 7 Find the domain of the following function.

$$q(y) = \frac{|y+1|}{(y-1)^2}$$

Exercise 8 Consider the following graph:



Is this the graph of a function and why?

Exercise 9 Let $f(x) = \sqrt{3-x}$ and $g(x) = \sqrt{x^2-1}$. Find $f+g$, $f-g$, fg, f/g and state their domains.

Exercise 10 Find the functions $f \circ g$, $g \circ f$, $f \circ f$, $g \circ g$ and their domains for the following:

(1) $f(x) = x^3 - 2$, $g(x) = 1 - 4x$

(2) $f(x) = \sin(x)$, $g(x) = x^2 + 1$

(3) $f(x) = \frac{x}{x+1}$, $g(x) = \sin(2x)$

Exercise 11 Find $f \circ g \circ h$ where

(1) $f(x) = |x-4|$, $g(x) = 2^x$, $h(x) = \sqrt{x}$.

(2) $f(x) = \tan(x)$, $g(x) = \frac{x}{x-1}$, $h(x) = \sqrt[3]{x}$.

Exercise 12 ($\star$) Suppose g is an even function and let $h = f \circ g$. Is h always an even function.

Exercise 13 ($\star$) Suppsoe g is an odd function and let $h = f \circ g$. Is h always an odd function?

Exercise 14 Let $f(x) = 2x^2 - 1$ and $g(x) = 4x^3 - 3x$. Show that $f \circ g = g \circ f$.

Exercise 15 Express the following in teh form of $f \circ g$.

(1) $F(x) = \cos^2(x)$.

(2) $q(x) = \sqrt[3]{\frac{x}{1+x}}$.

(3) $u(t) = \frac{\tan(t)}{1+\tan(t)}$.

Exercise 16 Find the exact trigonometric ratios for the angle whose radian measure is given:

(1) $\frac{4\pi}{3}$.

(2) -5π.

(3) $\frac{11\pi}{4}$.

i.e., find sin, cos, tan, csc, sec and cot.

Exercise 17 Find the remaining trigonometric ratios.

(1) $\cos(x) = -\frac{1}{3}$, where $\pi < x < \frac{3\pi}{2}$.

(2) $\csc(\theta) = -\frac{4}{3}$, where $\frac{3\pi}{2} < \theta < 2\pi$.

Exercise 18 Show that $\sin^2(x) - \sin^2(y) = \sin(x+y)\sin(x-y)$

Exercise 19 Prove that $\tan(x) + \tan(y) = \frac{\sin(x+y)}{\cos(x)\cos(y)}$.

Exercise 20 Use the law of exponents to rewrite and simplify the expressions:

(1) $x\left(3x^2\right)^3$.

(2) $\frac{x^{2n} \cdot x^{3n-1}}{x^{n+2}}$.

(3) $\frac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}}$.

Exercise 21 Given the following formula, determine whether they are one-to-one.

$$g(x) = \sqrt[3]{x}$$

Exercise 22 Find a formula for the inverse of the following function

$$f(x) = \frac{4x-1}{2x+3}$$