Midterm 2

Practice Exam

Due: 20 March 2020

Total Points: 100

(1) For now, there are no instructions until I know how well the practice crowdmark test works.

(2) There WILL be instructions on the actual midterm, so make sure to read them and follow them.

(3) Remember that if you help me test out the practice crowdmark test by Sunday (aka before Midterm 2) you get one extra point on your actual miderm.

Exercise 1 (10 points each) Find the following limits.

(1)
$$\lim_{x\to 0} (-2\,x+1)^{\left(\frac{1}{x}\right)}$$

(2)
$$\lim_{x\to 0} \frac{3^x x}{3^x-1}$$

(3)
$$\lim_{x\to 0} -\frac{x-e^x+1}{x^2}$$

Exercise 2 (10 points) Differentiate the following:

$$y = \left(x^4 + 4\right)^4 \left(x^2 + 2\right)^2$$

Exercise 3 (10 points) Suppose that s is a function of r. Find $\frac{d}{dr}s$ of the following function:

$$e^{\left(r^2 s\right)} = 12$$

Exercise 4 (20 points)

A person is walking along a straight path at a speed of 4 ft/s in the middle of the night. In order to help them see, their friend points a flashlight on them from across the 20 foot wide street. At what rate is the flaslight rotating when the person is 15 feet away from the nearest point to their friend?

Exercise 5 (5 points) Find a good approximation for

$$f(x) = x^{\frac{1}{3}} \text{ at } x = 1001$$

Exercise 6 (20 points) Using the techniques in class, sketch the curve of the following function.

$$f(x) = x^3 - 1$$

Things to include:

(1) Where does the function cross the x axis?

(2) What are the critical points? What are the inflection points?

(3) Where is the graph increasing/decreasing?

(4) Where is the graph concave up/down?

Exercise 1

Solution. (1) $\lim_{x\to 0}\left(-2\,x+1\right)^{\left(\frac{1}{x}\right)} = e^{(-2)}$

(2) $\lim_{x\to 0}\frac{3^x x}{3^x-1} = \frac{1}{\ln(3)}$

(3) $\lim_{x\to 0} -\frac{x-e^x+1}{x^2} = \frac{1}{2}$

□

Exercise 2

Solution.

$$y = \left(x^4+4\right)^4\left(x^2+2\right)^2$$

$$\frac{dy}{dx} = 16\left(x^4+4\right)^3\left(x^2+2\right)^2x^3 + 4\left(x^4+4\right)^4\left(x^2+2\right)x$$

□

Exercise 3

Solution.

$$\frac{d}{dr}s = -\frac{2\,s}{r}$$

□

Exercise 4

Solution. (1) Highlight - "straight", "4ft/s", "20 ft", "15 ft", "closest point", "speed rotating"

(2) Picture is a right triangle

(3) θ is angle; x is person walking path, y is distance across street.

(4) $\tan(\theta) = x/y$.

(5) $\sec^2(\theta)\frac{d\theta}{dt} = 1/y \cdot \frac{dx}{dt}$.

(6) $\frac{d\theta}{dt} = \frac{4}{20 \cdot \sec^2(\theta)} = \frac{\cos^2(\theta)}{5}$.

(7) When $x = 15$ we know $\cos(\theta) = \frac{20^2}{\sqrt{20^2+15^2}} = \frac{4}{5}$

(8) Solution: $\frac{d\theta}{dt} = \frac{\left(\frac{4}{5}\right)^2}{5} = \frac{16}{125} \approx 0.128$ rad/s.

□

Exercise 5

Solution.

$$L(x, 1000) = \frac{1}{300}x + \frac{20}{3} \qquad L(1001, 1000) = \frac{3001}{300}$$

□

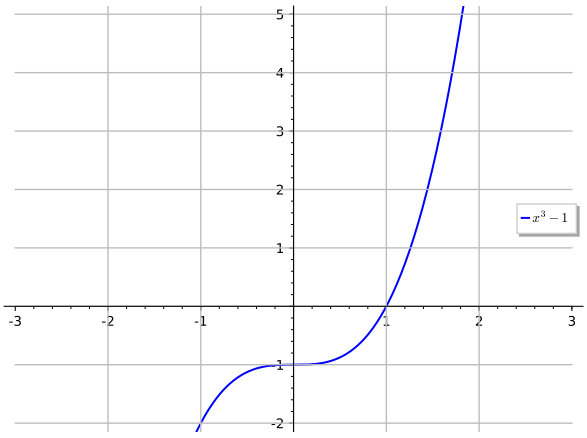
Exercise 6

Solution.

$f = x^3 - 1$ Roots: $\left[\left(\frac{1}{2}i\sqrt{3} - \frac{1}{2}, 1\right), \left(-\frac{1}{2}i\sqrt{3} - \frac{1}{2}, 1\right), (1, 1)\right]$.

$f' = 3\,x^2$ Crit points: $[(0, 2)]$.

$f'' = 6\,x$ Inf points: $[(0, 1)]$.



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