Midterm 1

Exercises

31 January 2020

Instructions:

- Duration of exam: 50 minutes.
- Only scientific (non-programmable) calculators are permitted.
- No notes/books are permitted
- You need to respond entirely on the pages given.
- The value of each question is noted to the right of the question.
- The solution to each question is worth 1 point; showing your work is worth the rest of the points.

1. (10 Points) Let $g(x) = \frac{x}{x-6}$ and $f(x) = x^2$.

(a) What is $(f \circ g)(x)$?

(b) What is the domain of $(f \circ g)(x)$? (why?)

(c) What is the range of $(f \circ g)(x)$? (why?)

(d) Is $(f \circ g)(x)$ even, odd, neither, or both? (why?)

2. (15 Points) Find the following limits. If the limit does not exist, explain why.

(a)

$$\lim_{x\to 0} -\frac{1}{x} + \frac{3}{\sqrt{x+9}x}$$

(b)

$$\lim_{x\to\infty} \frac{\sqrt{9\,x^2+3\,x+2}}{6\,x}$$

3. (15 Points) Find the following limits. If the limit does not exist, explain why.

(a)

$$\lim_{x \to 0^+} \frac{4 \sin(x) + 1}{x^7}$$

(b)

$$\lim_{x \to 0^-} \frac{4 \sin(x) + 1}{x^7}$$

(c)

$$\lim_{x \to 0} \frac{4 \sin(x) + 1}{x^7}$$

4. (10 Points) Find the following limit using the squeeze theorem.

$$\lim_{x \to +\infty} -\frac{\sin(x)^2 + 2}{2x - 5}$$

5. (15 Points) Show that the following function is continuous at $a = 2$ using the three requirements we went over in class.

$$f(x) = \frac{2\,x^2 + 5\,x + 4}{x^3 + 4\,x + 6}$$

(1) (a) $\frac{x^2}{(x-6)^2}$
(b) Domain: $\mathbb{R}\setminus\{6\}$
(c) Range: $\mathbb{R}_{\geq 0}$
(d) Neither even nor odd.
(2) (a) $-\frac{1}{18}$
(b) $\frac{1}{2}$
(3) (a) $+\infty$
(b) $-\infty$
(c) DNE
¡++¿
(4) 0
(5) (a) $f(2) = 1$
(b) $\lim_{x\to 2} f(x)$ exists
(c) $\lim_{x\to 2} f(x) = f(2)$.