Exercices 3 solutions

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Un exercice marqué du symbole ⋆ est considéré comme plus difficile et ne sera pas une question d'examen.

Exercice 1 Évaluer

$$\int_1^2 \frac{x^3+5\,x+4}{x^3}\,dx$$

Démonstration. 5 □

Exercice 2 Calculer les intégrales suivantes.

(1) $\int \sqrt{2\,x+3}\,dx$

(2) $\int -4\left(3\,x^2-5\right)^5 x\,dx$

(3) $\int \left(x^3-4\right)x\,dx$

(4) $\int \sec\left(x\right)^3 \tan\left(x\right)\,dx$

(5) $\int \frac{3\,x}{\sqrt{-x^2+1}}\,dx$

(6) $\int \frac{\sec\left(-\sqrt{x}+3\right)}{\sqrt{x}}\,dx$

(7) $\int \frac{1}{(4\,x-3)^2}\,dx$

(8) $\int \frac{12\,x^2}{x^3+8}\,dx$

(9) $\int \cos\left(2\,x\right)^4 \sin\left(2\,x\right)\,dx$

(10) $\int \frac{1}{\sqrt{x}\left(\sqrt{x}+5\right)}\,dx$

(11) $\int \frac{1}{\sin(2\,x)^2}+\sin\left(\frac{1}{2}\,x\right)\,dx$

(12) $\int \frac{3\,x+5}{x^2+1}\,dx$

(13)⋆ $\int \frac{\log(x)^2}{x}\,dx$

(14)⋆ $\int \frac{x^2-x-1}{x-1}\,dx$

(15)⋆ $\int \frac{2\left(4\,x^2-x-3\right)}{4\,x-3}\,dx$

(16)⋆ $\int \frac{\cos(x)}{\sin(x)^3+3\,\sin(x)^2+3\,\sin(x)+1}\,dx$

Démonstration. (1) $\frac{1}{3}\left(2\,x+3\right)^{\frac{3}{2}}$

(2) $-\frac{1}{9}\left(3\,x^2-5\right)^6$

(3) $\frac{1}{5}\,x^5-2\,x^2$

(4) $\frac{1}{3\,\cos(x)^3}$

(5) $-3\,\sqrt{-x^2+1}$

(6) $2\,\log\left(\sec\left(\sqrt{x}-3\right)+\tan\left(\sqrt{x}-3\right)\right)$

(7) $-\frac{1}{4\,(4\,x-3)}$

(8) $4\,\log\left(x^3+8\right)$

(9) $-\frac{1}{10}\,\cos\left(2\,x\right)^5$

(10) $2\,\log\left(\sqrt{x}+5\right)$
(11) $-\frac{1}{2\,\tan(2\,x)}-2\,\cos\left(\frac{1}{2}\,x\right)$
(12) $5\,\arctan\left(x\right)+\frac{3}{2}\,\log\left(x^2+1\right)$
(13) $\frac{1}{3}\,\log\left(x\right)^3$
(14) $\frac{1}{2}\,x^2-\log\left(x-1\right)$
(15) $x^2+x-\frac{3}{4}\,\log\left(4\,x-3\right)$
(16) $-\frac{1}{2\left(\sin(x)^2+2\,\sin(x)+1\right)}$

□

Exercice 3 Calculer les intégrales suivantes.

(1) $\int_0^1 3\left(x^3+1\right)^7 x^2\,dx$
(2) $\int_0^1 \frac{6\,x+1}{3\,x^2+x-1}\,dx$
(3) $\int_{\frac{1}{2}}^1 \frac{1}{(2\,x+1)^2}\,dx$
(4) $\int_0^{\frac{\pi}{2}} \frac{\cos(x)}{(\sin(x)+1)^2}\,dx$
(5) $\int_{-1}^0 e^{(3\,x+5)}\,dx$
(6) $\int_0^2 2\,(2\,x+1)e^{\left(x^2+x\right)}\,dx$
(7)⋆ $\int_0^1 \sqrt{-4\,x^4+8\,x^2}\,dx$

Démonstration. (1) $\frac{255}{8}$
(2) $\log\left(\frac{79}{16}\right)+2\,\log\left(2\right)$
(3) $\frac{1}{12}$
(4) $\frac{1}{2}$
(5) $\frac{1}{3}\,e^5-\frac{1}{3}\,e^2$
(6) $2\,e^6-2$
(7) $-\frac{4}{3}\,\sqrt{2}+\frac{2}{3}$

□

Exercice 4 Évaluer les intégrales suivantes.

(1) $\int_{-3}^3 \frac{x^9-x^3+x}{2\,x^6+5\,x^2+9}\,dx$
(2) $\int_{-1}^1 x^{15}\cos\left(x\right)+3\,x^2+\left(x^4+1\right)^{\frac{1}{7}}x\,dx$
(3) $\int \frac{1}{\sqrt{-x^2+2\,x}}\,dx$
(4) $\int \frac{1}{4\,x^2+12\,x+13}\,dx$
(5) $\int_0^2 \frac{x+4}{x^2+6\,x+10}\,dx$

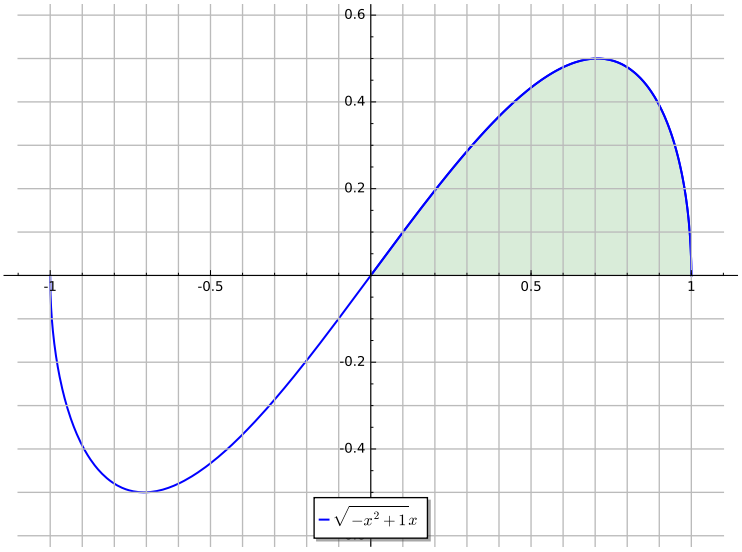
Démonstration. (1) 0
(2) 2
(3) $-\arcsin\left(-x+1\right)$
(4) $\frac{1}{4}\,\arctan\left(x+\frac{3}{2}\right)$

(5) $\arctan(5) - \arctan(3) + \frac{1}{2}\log(26) - \frac{1}{2}\log(10)$

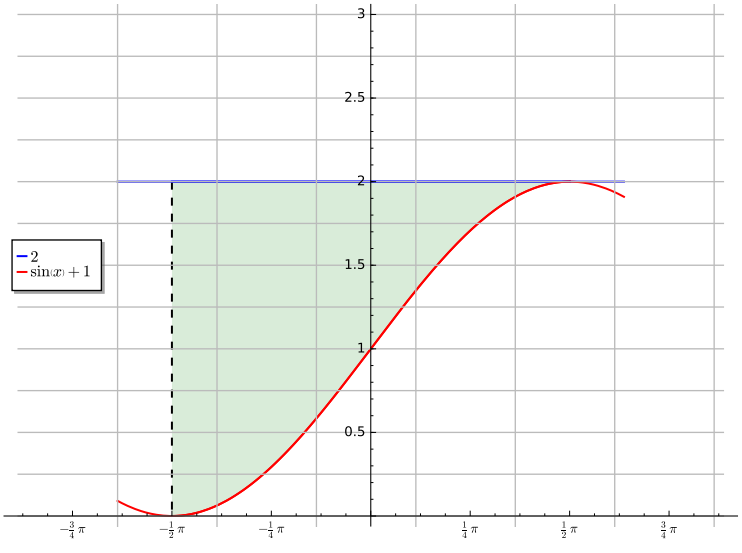
□

Exercice 5 Trouver l'aire de la surface.

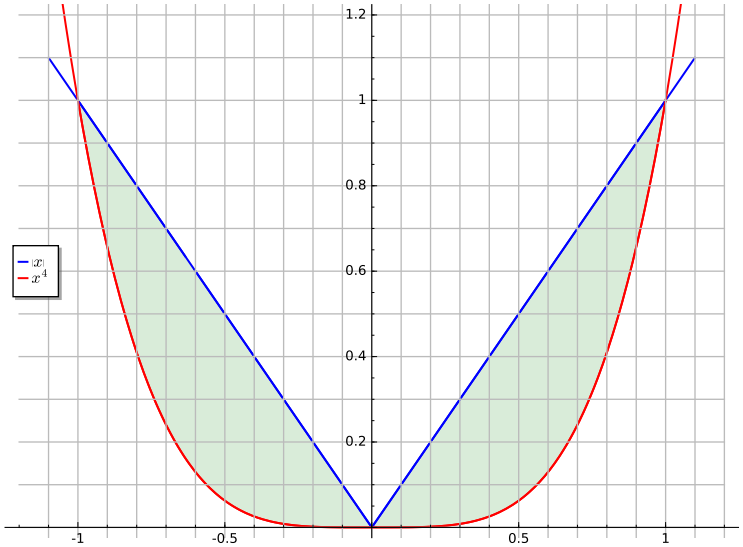
(1)



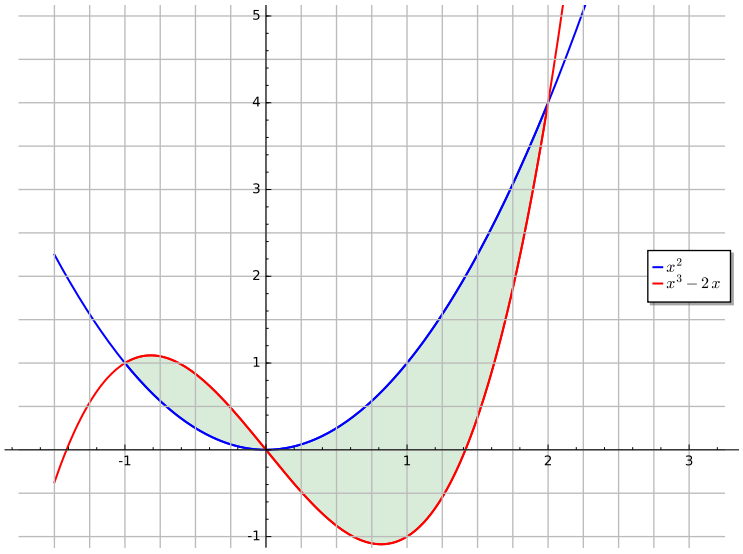
(2)



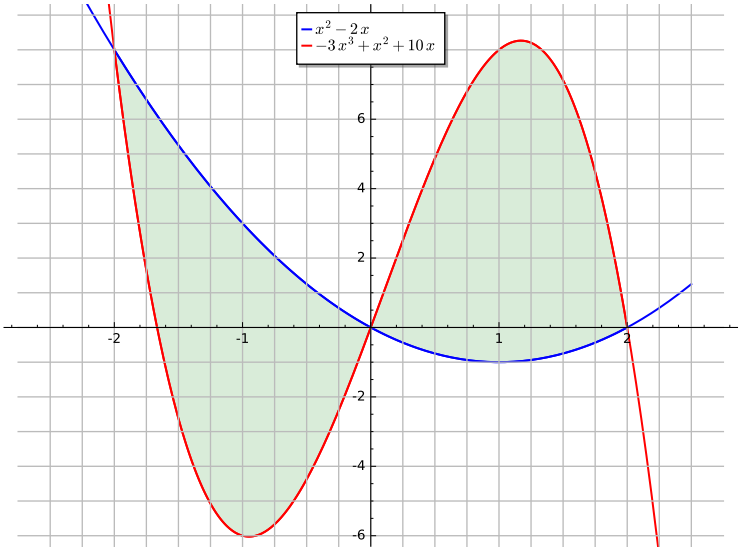
(3)



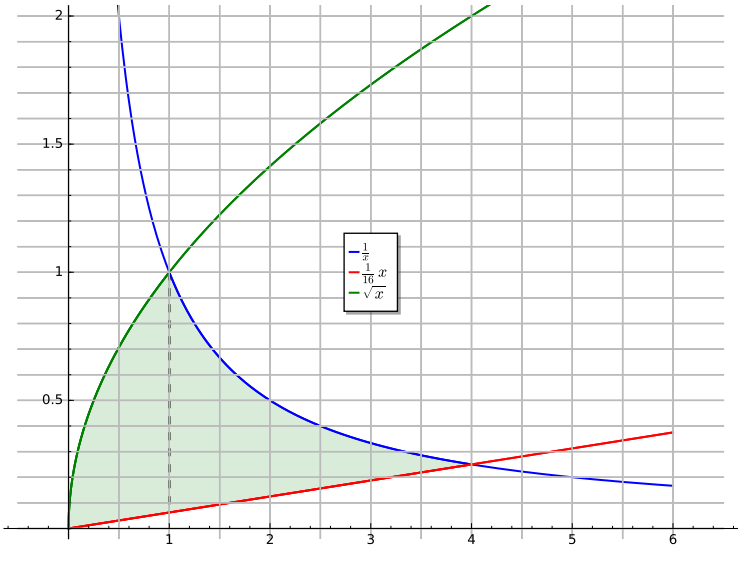
(4)



(5)



(6)



Démonstration. (1) $\frac{1}{3}$

(2) π

(3) $\frac{3}{5}$

(4) $\frac{37}{12}$

(5) 24

(6) $2\log(2) + \frac{1}{6}$

□