Week 1

Exercise 1 Suppose we have a bag where there are twice as many green marbles as red marbles. What is the proportion of green marbles as:

(1) a fraction

(2) a decimal

(3) a percentage

Exercise 2 Suppose we have the following:

How much wood could a woodchuck chuck if a wood chuck could chuck wood?

(Count each word separately even if it's the same word) What is the probability of the following events:

(1) A word has three letters.

(2) A word contains exactly two vowels.

(3) A word contains an "a" or an "i".

Exercise 3 Suppose there's a roulette table with 38 numbers: $1, 2, \ldots, 36, 0, 00$. The numbers $1 - 36$ are either red or black (half/half) and the 0 and 00 are both green. Suppose I bet on red and you bet on black. Once the bets are placed, the ball rolls and lands on a number.

(1) What is the probability that we both lose?

(2) What is the probability that we both win?

(3) What is the probability that at least one of us wins?

(4) What is the probability that at least one of us loses?

Exercise 4 Suppose we have a box with n tickets in it, each numbered from 1 to n (so that all numbers appear exactly once). If I draw a ticket at random, put it back in the box and pull out another random ticket again, what are the probabilities of the following events:

(1) The same number is pulled out twice.

(2) The two numbers are consecutive integers (*i.e.*, 3 gets pulled and then a 4 or 4 gets pulled and then a 3).

(3) The first number pulled is (strictly) less than the second number pulled.

If instead I draw a random ticket, leave it outside the box and pull out another random ticket, what are the probabilities of the same three events as above.

Exercise 5 Suppose I randomly shuffle a deck of 52 cards. I give you the first card and I take the second card. What are the probabilities of the following events:

(1) You received a 2.

(2) I received a 2.

(3) We both received a 2.

Exercise 6 Suppose I roll two 4-sided dice. Find the probabilities of the following events:

(1) The maximum of the two numbers rolled is less than or equal to 2.

(2) the maximum of the two numbers rolled is less than or equal to 3.

(3) The maximum of the two numbers equals exactly to 1.

(4) Repeat the previous question for 2, 3, and 4.

(5) What is the sum of the probabilities of the last two questions? Does that make sense?

Exercise 7 Say we're watching a horse race and the presenter says that "my little pony" has a $3:1$ odd in favour of winning. What is the probability that they will win?

Exercise 8 Say that our best friend is about to have a baby and you know that statistically roughly out of every 60 babies born, roughly 30 are assigned male at birth, 29 are assigned female at birth and 1 are assigned intersex at birth.

(1) What is the sample space?

(2) What is the probability that your best friend's baby is assigned female at birth?

(3) Is this a frequency or a subjective interpretation of probability?

Exercise 9 Prove that $\binom{n}{k} = \binom{n}{n-k}$

Exercise 10 Prove that $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Exercise 11 Given a normal 52 card deck.

(1) How many different 5 card hands are there?

(2) How many different 5 card hands do not have the number 2?

(3) How many different 5 card hands contain exactly one number 2?

(4) How many different 5 card hands contain all cards of the same suit?

Exercise 12 Suppose it's your birthday and your two best friends bought you a cake. You're about to cut it when your mom in the other room yells at you to save her a slice too! So you cut the cake into four different slices, one for each person. Obviously since it's your birthday you get the biggest slice which turns out to be the same size as both of your best friends' slices combined (your best friends get the same size slice)! The fourth slice for your mom turns out to be half the size of one of your best friend's slices. How much of the cake did you get?

Exercise 13 Suppose you and your friend are at a fair and you decide to join the raffle. You buy 4 tickets (numbered $71-74$) and your friend buys 5 tickets (numbered $89-93$). You know that there are 100 tickets in the raffle in total and the tickets are labelled (in order) starting from 1.

(1) What is the sample space?

(2) What subset represents the event "You won the raffle"?

(3) What subset represents the event "You or your friend won the raffle"?

(4) What subset represents the event "You and your friend won the raffle"?

(5) What subset represents the event "The number drawn was 1 away from either you or your friend's tickets, but not exact with any ticket."?

Example 14 Let $\Omega = \{0, 1, 2\}$ be the sample space for flipping a coin twice where each outcome shows how many heads you got. Which of the following is an event in this space? If it's an event, give its subset representation and if it's not an event, explain why.

(1) The coin lands on heads twice.

(2) The coin lands on heads at least once.

(3) The coin lands on heads and then lands on tails.

(4) The coin lands on heads once and lands on tails once.

Extra: Why is this question ok when it looks like one of the common student mistakes found in class?

Exercise 15 Suppose we have the following:

How much wood could a woodchuck chuck if a wood chuck could chuck wood?

(1) What is the distribution of the length of the words?

(2) What is the distribution of the number of vowels per word?

(Count each word separately even if it's the same word)

Exercise 16 Show that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Exercise 17 Suppose we have the events A, B, and Z where

$$P(A) = 0.8 \quad P(B) = 0.4 \quad P(Z) = 0.3$$
$$P(A \cap B) = 0.2 \quad P(A \cap Z) = 0.2 \quad P(B \cap Z) = 0.1$$
$$P(A \cap B \cap Z) = 0$$

Find the following probabilities:

(1) $P(A \cup B)$

(2) $P(A \cup B \cup Z)$

(3) $P(A \cap B^c \cap Z^c)$