Week 1

6–10 Jan 2020

Like any first class, we go over the basics today.

My info:

- __________
- __________________________

Office Hours:

- ___________________
- ___________________
- __________________
- _______________________

Grading:

- _______________________
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Other:

- ____
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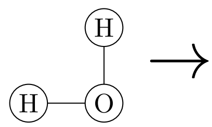
What is calculus good for? Calculus is used throughout the world for many things. Most people know that calculus is used in physics, chemistry and economics, but it's also used in construction, medicine, geology. If you plan on working in any of the sciences, then you'll likely be using calculus.

1.1 Functions - §1.1 – 1.3

A function is nothing more than a rule where you give it something and get back something else. As an example:



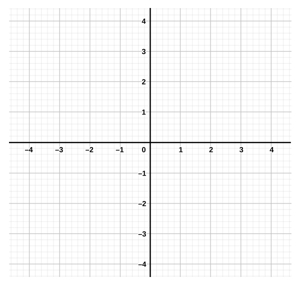
More explicitly, a *function* f is a rule that assigns to each x in a set D to *exactly* one element ________ , in a set E. This is normally denoted ____________________ where D is called the *domain* and E is called the *codomain*. The *range* of f is the set of all possible values of $f(x)$ for every x in D.

Exercise 1.1 Create your own function that does not use any numbers nor atoms! When finished, compare with your neighbor.

Example 1.2 Let's look at some examples.

(1) A *linear function* is a function whose graph is a line (*e.g.* functions of the form $f(x) = mx + b$. Let f be the linear function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) =$ ____.

Domain: ___ Codomain: ___ Range: ___

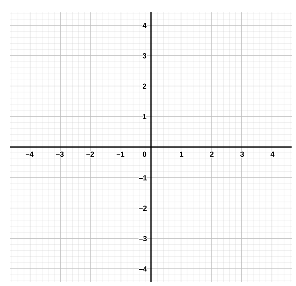


(2) A *polynomial* f is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0$$

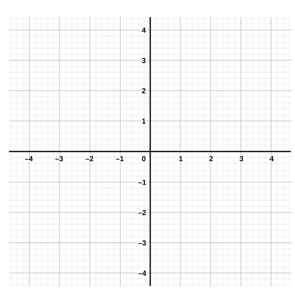
where $a_0, \ldots, a_n$ are known as the *coefficients* and, if $a_n \neq 0$, then f is said to have *degree* n.
Let $f : \mathbb{R} \to \mathbb{R}$ be the polynomial of degree 2 such that $f(x) =$ ___________.

Domain: ___ Codomain: ___ Range: _______



(3) Let f be the function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) =$ ______.

Domain: ___ Codomain: ___ Range: ___



(4) A *rational function* f is a ratio of two polynomials:

$$f(x) = \frac{P(x)}{Q(x)}.$$

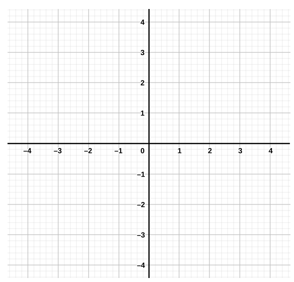
Let f be the rational function

$f(x) =$ ____________________

Domain: ________ Range: ___

Why am I not asking for the codomain?

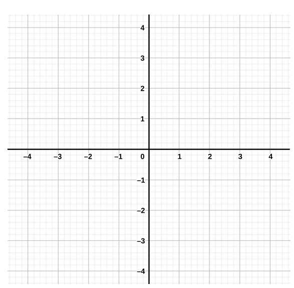
____



(5) *Piecewise defined function:* is a function which has different formulas in different parts of their domains. Let f be the piecewise defined function $f : \mathbb{R} \to \mathbb{R}$ such that

$f(x) =$ ____________________

Domain: ___ Codomain: ___ Range: ______



An ________ function is a function f that satisfies $f(-x) = f(x)$. An ______ function is a function f that satisfies $f(-x) = -f(x)$.

Example 1.3 Let $f : \mathbb{R} \to \mathbb{R}$ be the function $f(x) =$ ____________. Is it even or odd? ______

Example 1.4 Let $f : \mathbb{R} \to \mathbb{R}$ be the function $f(x) = \sin(x)$ and $g : \mathbb{R} \to \mathbb{R}$ be the function $g(x) = \cos(x)$. Which is even and which is odd?

Even: _____ Odd: _____

Exercise 1.5 With a person next to you: Give an example of a function that is neither even nor odd:

Exercise 1.6 With a person next to you: Give an example of a function that is both even and odd:

Recall that $[a, b]$ is a *closed interval* of $\mathbb{R}$ and (a, b) is an *open interval* of $\mathbb{R}$. A function f is called ________________ on an interval I if:

$$f(x) < f(y) \text{ for every } x < y \text{ in } I.$$

A function f is called ________________ on an interval I if:

$$f(x) > f(y) \text{ for every } x < y \text{ in } I.$$

Let f and g be two functions. Then

$(f + g)(x) =$ __________________ $(f - g)(x) =$ __________________

$(fg)(x) =$ ______________ $\left(\frac{f}{g}\right)(x) =$ ________

Example 1.7 Let $f(x) =$ ____ and $g(x) =$ ________.

$(f + g)(x) =$ ________________ $(f - g)(x) =$ ________________

$(fg)(x) =$ ____________ $\frac{f}{g}(x) =$ _________

The ___________________ of two functions is the *composite function* $f \circ g$ where

$$f \circ g(x) = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

The order matters for composition, just like subtraction and division!

Example 1.8 Let $f(x) =$ ____ and $g(x) =$ ________.

$(f \circ g)(x) =$ ____________ $(g \circ f)(x) =$ __________.

Exercise 1.9 With the person next to you: Let $f(x) =$ ________ and $g(x) =$ ________.

$(f \circ g)(x) =$ ________________ $(g \circ f)(x) =$ ___________________.

1.2 Exponential functions - §1.4

An *exponential function* is a function of the form

$$f(x) = \underline{\qquad}$$

where b is a positive constant (*i.e.*, $b \in \mathbb{R}^+$).

How do we calculate b^x for every $x \in \mathbb{R}$?

x	b^x	Example
Integer ($\mathbb{Z}$)	b^x	
Rational ($\mathbb{Q}$)	$b^{p/q} = \sqrt[q]{b^p}$	
Irrational ($\mathbb{R} \backslash \mathbb{Q}$)	Estimate using graph	

Example 1.10 Let $f(x) =$ ____ and let $g(x) =$ ____.

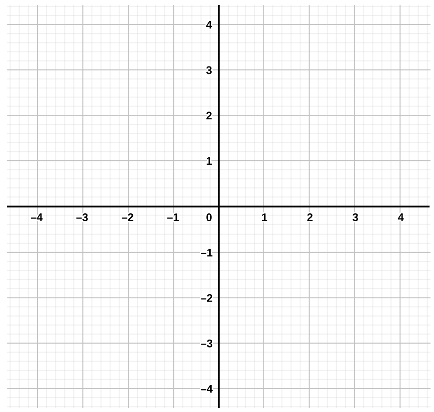
$(f \circ g)(x) =$ _______ $(g \circ f)(x) =$ ________

Are these the same function? (*i.e.*, is $f \circ g = g \circ f$) ____ .

1.3 Trigonometric functions - Appendix D

If any part of this section does not make sense, please raise your hand and force me to stop. Every country teaches trigonometry differently and at different times, so I literally have no idea what everyone knows/doesn't know.

Aram's favorite way to view trig functions:



This helps us calculate all the other trig functions as well:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}, \ \csc(\theta) = \frac{1}{\sin(\theta)}, \ \sec(\theta) = \frac{1}{\cos(\theta)}, \ \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Although we had the radius be equal to 1, we can alter the radius! Then we need a new method to solve for our trig functions. This is where an acronym comes in: __________________ . This then gives us the following ways to calculate all the trig functions.

There are a few trig functions that are worthwhile to know how to quickly calculate.

Using the triangles, we can also come up with some trig identities.

(1) ________________________________

(2) ________________________________

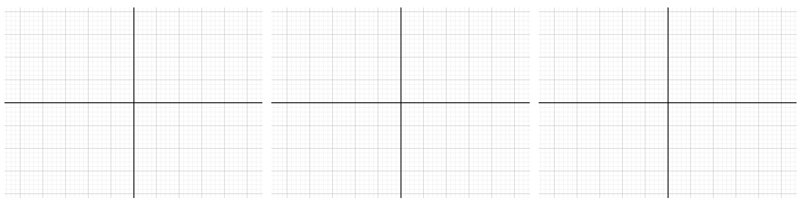
(3) ________________________________

There are also addition formulas which I'll state, but won't go over to much.

(1) ____

(2) ____

We end this part by giving the graphs of all three functions:



1.4 Inverse functions - §1.5

A function allows us to map some number a to some number b using a formula, but what if we want to go backwards? How do we find a function that allows us to go from b to a, and is this even possible?

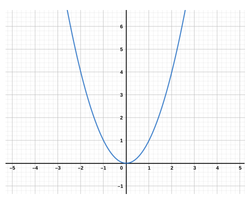
Example 1.11 Let's try an easy example. Let f be the function $f(x) =$ ________. To go backwards we would need to use the formula ________ .

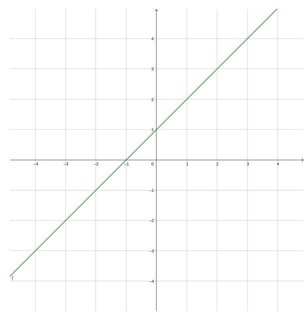
But this doesn't always work!

Example 1.12 Let f be the function $f(x) = x^2$. If I know $f(x) = 4$, what is x?

To be able to define an inverse, it seems as though we need a rule on our functions. A function f is *one-to-one* if

If we have a graph there is a nice way to test this:





Proposition 1.13 *A function is one-to-one if and only if*

Definition 1.14 Let $f : A \to B$ be a one-to-one function with range B. Then its inverse function is the function $f^{-1} : B \to A$ with range A and is defined by

This means that a function being one-to-one is dependent on its domain.

Example 1.15 For example, we saw $f : \mathbb{R} \to \mathbb{R}^+$ where $f(x) = x^2$ is not one-to-one. But if we change our function's domain so that ______________________ then $f(x) = x^2$ is one-to-one. Its inverse function is ______ .