

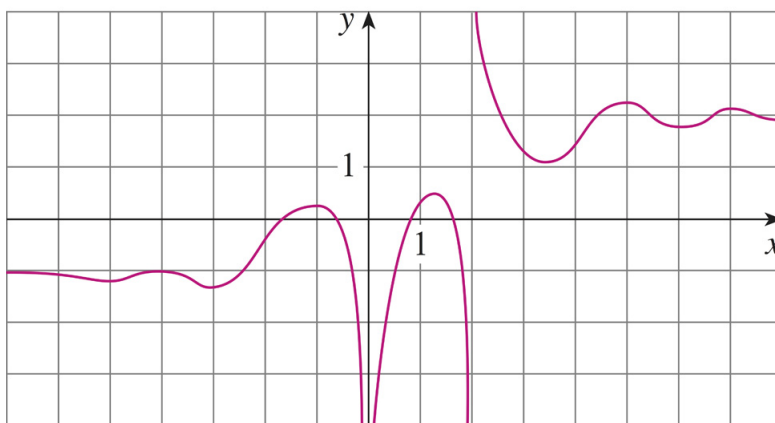
# Homework 4 solutions

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An exercise marked with the symbol  $\star$  is considered more difficult and will not be an exam question.

**Exercise 1** Given the following graph of the function  $f$ :



Find the following limits:

(1)  $\lim_{x \rightarrow \infty} f(x)$

(2)  $\lim_{x \rightarrow -\infty} f(x)$

(3)  $\lim_{x \rightarrow 0} f(x)$

(4)  $\lim_{x \rightarrow 2^+} f(x)$

(5)  $\lim_{x \rightarrow 2^-} f(x)$

*Solution.* (1) 2

(2)  $-1$

(3)  $-\infty$

(4)  $\infty$

(5)  $-\infty$

□

**Exercise 2** Find the following limits

(1)

$$\lim_{x \rightarrow \infty} \frac{1 - x^2}{x^3 - x + 1}$$

(2)

$$\lim_{x \rightarrow -\infty} \frac{4x^3 + 6x^2 - 2}{2x^3 - 4x + 5}$$

(3)

$$\lim_{t \rightarrow \infty} \frac{t - t\sqrt{t}}{2t^{3/2} + 3t - 5}$$

(4)

$$\lim_{x \rightarrow \infty} \frac{x^2}{\sqrt{x^4 + 1}}$$

(5)

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{1 + 4x^6}}{2 - x^3}$$

(6)

$$\lim_{x \rightarrow \infty} \frac{x + 3x^2}{4x - 1}$$

(7)

$$\lim_{x \rightarrow -\infty} \sqrt{4x^2 + 3x} + 2x$$

(8)

$$\lim_{x \rightarrow \infty} \sqrt{x^2 + 1}$$

(9)

$$\lim_{x \rightarrow \infty} e^{-x} + 2 \cos(3x)$$

(10)

$$\lim_{x \rightarrow -\infty} \frac{1 + x^6}{x^4 + 1}$$

(11)

$$\lim_{x \rightarrow \infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$$

(12)

$$\lim_{x \rightarrow \infty} \frac{\sin^2(x)}{x^2 + 1}$$

(13)

$$\lim_{x \rightarrow 0^+} \tan^{-1}(\ln(x))$$

(14)

$$\lim_{x \rightarrow \infty} \ln(2 + x) - \ln(1 + x)$$

*Solution.* (1) 0

(2) 2

(3)  $-\frac{1}{2}$ 

(4) 1

(5) 2

(6)  $\infty$ (7)  $-\frac{3}{4}$ (8)  $\infty$ 

(9) DNE

(10)  $\infty$ 

(11) 1

(12) 0 (Hint: Squeeze theorem)

(13)  $\frac{\pi}{2}$ 

(14) 0

□

**Exercise 3** Find  $f'(a)$  for the following functions.

(1)  $f(t) = 2t^3 + t$

(2)  $f(x) = x^{-2}$

(3)★  $f(x) = \frac{4}{\sqrt{1-x}}$

*Solution.* (1)  $6a^2 + 1$ 

(2)  $-\frac{2}{a^3}$

(3)  $\frac{2}{(1-a)^{3/2}}$

□

**Exercise 4** Find the derivative of the following functions using the definition of the derivative.

(1)  $f(x) = 4 + 8x - 5x^2$

(2)  $g(t) = \frac{1}{\sqrt{t}}$

(3)★  $f(x) = \frac{x^2-1}{2x-3}$

(4)  $f(x) = x^{3/2}$

*Solution.* (1)  $f'(x) = 8 - 10x$

(2)  $g'(t) = -\frac{1}{2t^{3/2}}$

(3)  $f'(x) = \frac{2x^2-6x+2}{(2x-3)^2}$

(4)  $\frac{3}{2}x^{1/2}$

□

**Exercise 5** Given the following functions, find the first, second, third and fourth derivatives.

(1)  $f(x) = x^3 - 3x$

(2)  $f(x) = x^5 + x^4 + x^3 + x^2 + x$

*Solution.* (1) (a)  $f'(x) = 3x^2 - 3$

(b)  $f''(x) = 6x$

(c)  $f'''(x) = 6$

(d)  $f^{(4)}(x) = 0$

(2) (a)  $f'(x) = 5x^4 + 4x^3 + 3x^2 + 2x + 1$

(b)  $f''(x) = 20x^3 + 12x^2 + 6x + 2$

(c)  $f'''(x) = 60x^2 + 24x + 6$

(d)  $f^{(4)}(x) = 120x + 24$

□