

## Homework 2

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An exercise marked with the symbol  $\star$  is considered more difficult and will not be an exam question.

**Exercise 1** Find the exact value of each expression.

(1)  $\log_5\left(\frac{1}{125}\right)$

(2)  $\ln\left(\frac{1}{e^2}\right)$

(3)  $e^{-\ln(2)}$

(4)  $e^{\ln(\ln(e^3))}$

**Exercise 2** Let  $f(x) = \ln(x - 1) - 1$ . Sketch the graph of  $f$  and find the following:

- The domain
- The range
- The  $x$ -intercept

**Exercise 3** Solve each equation for  $x$ :

(1)  $\ln(x^2 - 1) = 3$

(2)  $e^{2x} - 3e^x + 2 = 0$

(3)  $\ln(\ln(x)) = 1$

(4)  $e^{ax} = Ce^{bx}$  where  $a \neq b$ .

**Exercise 4** Find the exact value of each expression.

(1)  $\tan^{-1}(\sqrt{3})$

(2)  $\arctan(-1)$

(3)  $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

(4)  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(5)  $\arcsin(\sin(5\pi/4))$

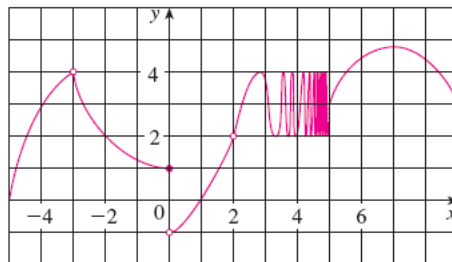
(6)  $\cos(2 \sin^{-1}(\frac{5}{13}))$

**Exercise 5** Explain in your own words what is meant by the equation:

$$\lim_{x \rightarrow 2} f(x) = 5.$$

Is it possible for this statement to be true and yet  $f(2) = 3$ ?

**Exercise 6** Suppose the function  $f$  has the following graph:



State the value of each quantity if it exists and if it doesn't exist, explain why.

(1)

$$\lim_{x \rightarrow -3^-} f(x)$$

(2)

$$\lim_{x \rightarrow -3^+} f(x)$$

(3)

$$\lim_{x \rightarrow -3} f(x)$$

(4)  $f(-3)$

(5)

$$\lim_{x \rightarrow 0^-} f(x)$$

(6)

$$\lim_{x \rightarrow 0^+} f(x)$$

(7)

$$\lim_{x \rightarrow 0} f(x)$$

(8)  $f(0)$

(9)

$$\lim_{x \rightarrow 2} f(x)$$

**Exercise 7** Sketch the graph of an example of a function  $f$  that satisfies all the given conditions.

- $\lim_{x \rightarrow 0} f(x) = 1$
- $\lim_{x \rightarrow 3^-} f(x) = -2$
- $\lim_{x \rightarrow 3^+} f(x) = 2$
- $f(0) = -1$
- $f(3) = 1$

**Exercise 8** Guess the value of the limit (if it exists) by evaluating the function at the given numbers using a calculator.

(1)

$$\lim_{x \rightarrow -3} \frac{x^2 - 3x}{x^2 - 9}$$

for

$$x = -2.5, -2.9, -2.95, -2.99, -2.999, -2.9999, -3.5, -3.1, -3.01, -3.001, -3.0001$$

(2)

$$\lim_{h \rightarrow 0} \frac{(2+h)^5 - 32}{h}$$

for

$$x = \pm 0.5, \pm 0.1, \pm 0.01, \pm 0.001, \pm 0.0001$$

**Exercise 9** Use a table of values to estimate the value of the limit for the following limits.

(1)

$$\lim_{p \rightarrow -1} \frac{1 + p^9}{1 + p^{15}}$$

(2)

$$\lim_{t \rightarrow 0} \frac{5^t - 1}{t}$$

(3)

$$\lim_{x \rightarrow 0^+} x^2 \ln(x)$$

**Exercise 10** Determine the infinite limit in the following limits.

(1)

$$\lim_{x \rightarrow 5^-} \frac{x+1}{x-5}$$

(2)

$$\lim_{x \rightarrow 3^-} \frac{\sqrt{x}}{(x-3)^5}$$

(3)

$$\lim_{x \rightarrow 0^+} \ln(\sin(x))$$

(4)

$$\lim_{x \rightarrow \pi^-} \cot(x)$$

(5)

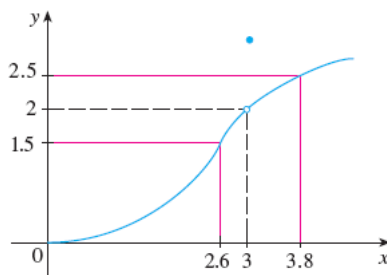
$$\lim_{x \rightarrow 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4}$$

(6)

$$\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \ln(x) \right)$$

**Exercise 11** Use the given graph of  $f$  to find a number  $\delta$  such that

$$\text{if } 0 < |x - 3| < \delta \text{ then } |f(x) - 2| < 0.5$$



**Exercise 12** Prove the following limits using an  $\varepsilon$ ,  $\delta$  proof.

(1)

$$\lim_{x \rightarrow 10} \left( 3 - \frac{4}{5}x \right) = -5$$

(2)

$$\lim_{x \rightarrow -1.5} \frac{9 - 4x^2}{3 + 2x} = 6$$

(3)

$$\lim_{x \rightarrow a} c = c$$