

# Common Mistakes

## Midterm 1

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These were the most common mistakes from the first exam, in no particular order. I will (and have) removed points for people making these mistakes.

- (1) **Dividing by 0:** You can not divide by 0.  $\frac{1}{0}$  does not equal  $\infty$ , it is undefined. Also,  $\frac{0}{0}$  is not equal to 1, it is undefined. You can never, under any circumstances divide by 0. If you don't know why, ask me.
- (2) **Dividing by  $\infty$ :** You can also never divide by  $\infty$  because  $\infty$  is not a number.  $\frac{1}{\infty}$  is not equal to 0, because dividing by something that is not a number is not defined. Similarly  $\frac{\infty}{\infty}$  is not equal to 1, nor is it equal to  $\infty$ , because, again, it is not defined. You can never divide by something which is not a number. (It's like trying to drink water by sending an email).
- (3) **Taking the limit:** If you take the limit, make sure to remove the limit sign. For example, the following is wrong as a method of proof:

$$\lim_{x \rightarrow 1} x = \lim_{x \rightarrow 1} 1$$

Although the two are equal at  $x = 1$ , the two are not equal in other places and so you need to be extremely careful! The correct way of doing this is to remove the limit sign after you have taken the limit:

$$\lim_{x \rightarrow 1} x = 1$$

This helps with 2 things: (1) I know *when* you took the limit and (2) that you understand equality.

- (4) **Using  $\frac{1}{x^r}$  wrong:** I saw a lot of people who did something like this:

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{x+1}{x} &= \lim_{x \rightarrow 1} \frac{x+1}{x} \cdot \frac{1/x}{1/x} \\ &= \lim_{x \rightarrow 1} \frac{1+1/x}{1} \\ &= \frac{1+0}{1} \end{aligned}$$

The error is the following:

$$\lim_{x \rightarrow 1} \frac{1}{x} \neq 0$$

The reason this is wrong is because the limit is going to 1! If I plug in 1 (which is not a method of proof, see below), I would notice that I should get 1, not 0. Basically, a lot of people are trying to use the following rule:

$$\lim_{x \rightarrow \infty} \frac{1}{x^r} = 0$$

when the limit is NOT going to infinity. This rule is ONLY defined when  $x$  goes to infinity. If we are taking the limit somewhere else, you need to avoid this equality at all costs because it is always (yes, always) wrong.

- (5) **Denominator is 0:** A lot of people mentioned that if the denominator is equal to 0 then the limit is not defined. This is very much not true. As an example:

$$\lim_{x \rightarrow 0} \frac{x^2}{x}$$

Notice that the denominator IS 0 since  $\frac{x^2}{x} = \frac{0}{0}$ , which means our function is not defined at  $x = 0$ . But just because the function is not defined, does NOT mean the limit is not defined. Here we can divide  $x$  to get:

$$\lim_{x \rightarrow 0} \frac{x^2}{x} = \lim_{x \rightarrow 0} \frac{x}{1} = \lim_{x \rightarrow 0} x = 0$$

Notice how the limit is defined!

- (6) **Exponential is not the same as natural log:** I saw a lot of people do something like the following:

$$e^x = xe$$

This is not true. The only time we can bring the exponent down is in logarithms, where we have the exponent INSIDE the logarithm. Example:

$$\log_2(t^5) = 5 \log_2(t)$$

BUT if the exponent is outside, we can't!

$$\log_2(t)^5 \neq 5 \log_2(t)$$

This is because the exponent of 5 is not applied to the  $t$ , instead it is applied to the logarithm, which changes everything! The reason we can put the 5 in front of the logarithm is because of the following rule:

$$\log(x \cdot y) = \log(x) + \log(y)$$

In other words:

$$\log(x^5) = \log(x \cdot x \cdot x \cdot x \cdot x) = \log(x) + \log(x) + \log(x) + \log(x) + \log(x) = 5 \log(x)$$

- (7) **Plugging in arbitrary numbers:** You can not take limits by plugging in numbers close to the limit. If you are taking the limit at  $x = 0$ , you cannot plug in 0.1 and say that it's going to be something similar. Plugging in numbers is ONLY a tool to help you figure out what the limit *should* be, not what the limit actually is. You need to use other methods to find the limit other than plugging in.
- (8) **Going to fast on a problem:** A lot of people went fast on a few questions and therefore lost points because they didn't explain themselves well. It's better to go slow and finish 80% of an exam perfectly than go fast and finish 100% of an exam sloppily. Since I grade heavily on explanation, writing sloppily will make you lose much more points giving you less than 80%. So please, slow down, think about the problem, and then solve it slowly.