

# Midterm 1

Exercises

31 January 2020

## **Instructions:**

- Duration of exam: 50 minutes.
- Only scientific (non-programmable) calculators are permitted.
- No notes/books are permitted
- You need to respond entirely on the pages given.
- The value of each question is noted to the right of the question.
- The solution to each question is worth 1 point; showing your work is worth the rest of the points.

1. (10 Points) Let  $g(x) = \frac{x}{x-6}$  and  $f(x) = x^2$ .
- (a) What is  $(f \circ g)(x)$ ?
  - (b) What is the domain of  $(f \circ g)(x)$ ? (why?)
  - (c) What is the range of  $(f \circ g)(x)$ ? (why?)
  - (d) Is  $(f \circ g)(x)$  even, odd, neither, or both? (why?)

**2. (15 Points)** Find the following limits. If the limit does not exist, explain why.

(a)

$$\lim_{x \rightarrow 0} -\frac{1}{x} + \frac{3}{\sqrt{x+9x}}$$

(b)

$$\lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + 3x + 2}}{6x}$$

**3. (15 Points)** Find the following limits. If the limit does not exist, explain why.

(a)

$$\lim_{x \rightarrow 0^+} \frac{4 \sin(x) + 1}{x^7}$$

(b)

$$\lim_{x \rightarrow 0^-} \frac{4 \sin(x) + 1}{x^7}$$

(c)

$$\lim_{x \rightarrow 0} \frac{4 \sin(x) + 1}{x^7}$$

4. (10 Points) Find the following limit using the squeeze theorem.

$$\lim_{x \rightarrow +\infty} \frac{\sin(x)^2 + 2}{2x - 5}$$

5. (15 Points) Show that the following function is continuous at  $a = 2$  using the three requirements we went over in class.

$$f(x) = \frac{2x^2 + 5x + 4}{x^3 + 4x + 6}$$